IN-HAND MANIPULATION USING EXTRINSIC DEXTERITY AND SIMPLE ASSISTIVE TOOLS

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ABSTRACT

Extrinsic dexterity consists in using external forces such as gravitational forces, inertia induced forces and contact forces to manipulate objects. This approach can be used to perform in-hand manipulation with grippers that have limited intrinsic dexterity. This paper investigates the use of extrinsic dexterity for the in-hand robotic manipulation of objects. A brief state of the art of the use of extrinsic dexterity is first provided. Then, a general dynamic model is developed to characterize the motion of a grasped object with respect to a robotic gripper. The model includes gravitational forces, inertia forces and contact forces (including friction forces). In order to demonstrate the application of the model, two simple one-degree-of-freedom situations are considered in which inertial forces and contact forces are used to reorient an object grasped by a two-finger gripper. Simulation results are provided to validate the results predicted by the model. Finally, the design of an assistive tool (also referred to as a 'left hand') is considered and a simple situation is described in order to explain the optimization procedure.

Keywords: robotic manipulation; extrinsic dexterity; simulation.

UTILISATION DE LA DEXTÉRITÉ EXTRINSÈQUE ET D'OUTILS EXTERNES SIMPLES POUR LA MANIPULATION D'OBJETS

RÉSUMÉ

La dextérité extrinsèque consiste à utiliser les efforts externes comme la gravité, les efforts inertiels ou les forces de contact pour manipuler des objets. Cette approche permet d'effectuer des manipulations dextres avec un préhenseur dont la dextérité intrinsèque est limitée. Cet article étudie l'utilisation de la dextérité extrinsèque pour la manipulation d'objets. Une brève revue de l'état de l'art sur la dextérité extrinsèque est d'abord présentée. Ensuite, un modèle dynamique général est développé pour caractériser le mouvement d'un objet saisi par un préhenseur. Le modèle tient compte des efforts dus à la gravité, aux effets inertiels ainsi qu'aux forces de contact, incluant le frottement. Afin de démontrer l'utilisation du modèle, deux situations simples de mouvement à un degré de liberté sont analysées. Dans ces situations, les efforts inertiels et les contacts sont utilisés pour réorienter un objet saisi par un préhenseur simple. Des résultats de simulation sont montrés afin de valider le modèle. Finalement, la conception d'un outil d'assistance (main gauche) est considérée et une situation simple est décrite afin d'expliquer la procédure d'optimisation de l'outil.

Mots-clés : manipulation robotique ; dextérité extrinsèque ; simulation.

1. INTRODUCTION

1.1. Robotic manipulation

The manipulation of an object grasped by a robot hand, also called in-hand manipulation, consists in repositioning the object with respect to the hand. In-hand manipulations are important since the desired pose of an object with respect to the hand may change. Also, it alleviates constraints on the grasping task, since grasping can then be carried out without taking into account the desired final grasp. These manipulations can be achieved by using the internal hand capabilities or by exploiting external resources. These two approaches are respectively referred to as intrinsic and extrinsic dexterity. The former method requires a hand with high dexterity, implying numerous degrees of freedom (DoF). Human-inspired hands are examples of the application of this method. This kind of manipulation is still an active field of research, for which kinematic and dynamic models have been developed [1]. However, many industrial grippers (purposely) do not possess such a high dexterity. Therefore, these manipulation techniques cannot be applied. On the other hand, extrinsic dexterity uses external resources like gravity, object inertia or contacts with the environment, as described in [2]. Therefore, simpler gripper architectures can be considered.

Robotic manipulation methods can be classified considering the constraint type induced by the end-effector on the object. The condition of constraints can be expressed as an equation $f(\cdot)$ connecting the coordinates of the object $\mathbf{r}$ (and possibly the time t). The first kind of constraints is referred to as bilateral and is expressed as

$$f(\mathbf{r},t) = 0 \,. \tag{1}$$

This type of constraint removes directly a DoF (*e.g.*, a simple pendulum). When all the constraints applied to an object are defined as bilateral, the manipulation is called prehensile. These manipulations are carried out when using intrinsic dexterity with form or force closures [3]. The second kind of constraints is referred to as unilateral and is expressed as

$$f(\mathbf{r},t) \geq 0 \,. \tag{2}$$

This constraint restrains the object to move in a delimited region of space. When the object is constrained with unilateral constraints, the manipulation is said to be nonprehensile. An example of such manipulations is the case of an object laying on an end-effector and which can be manipulated by the motion of the end-effector. It can be noted that other classifications based on the constraint types exist but are not presented here, see [4] for more details on such classifications.

Manipulation tasks can also be defined by the type of analysis needed for the implementation [5]. Four types of tasks can be defined, namely *i*) the kinematic manipulation where only kinematic analysis is carried out (*e.g.*, for some pick and place tasks); *ii*) the static manipulation where static forces such as friction are also considered (*e.g.*, for some robotic grippers); *iii*) the quasi-static manipulation which allows the study of objects slipping on a surface for instance, and finally; *iv*) dynamic manipulations where forces related to acceleration are involved (*e.g.*, stabilizing an object rolling on a plane). This last type of analysis coupled with the constraints defining nonprehensile manipulation (Eq. (2)) is referred to as nonprehensile dynamic manipulation. This manipulation type is still an active research topic and more details can be found in [6]. In this field, tasks are often divided into simpler sub-tasks. These sub-tasks are referred to as primitives of the nonprehensile dynamic manipulation. Different primitives exist such as throw, grip, hit, push, slip or roll.

In this paper, the situation in which an object is grasped by a two-finger gripper is considered. Therefore, it is constrained in a plane defined by the fingers: the object has three potential DoFs with respect to the gripper and the constraints applied to the object are bilateral. However, there is a high similarity with the slip primitive of nonprehensile manipulation. Moreover, we can enhance the manipulation capabilities of the two-finger gripper thanks to extrinsic dexterity. We then desire a model that can define the motion of an object when it is subjected to gravity, accelerations of the gripper or external contacts. Thus, this model requires a complex dynamic analysis of the object. A further investigation of the state of the art on extrinsic dexterity is then carried out.

1.2. Extrinsic dexterity

As mentioned above, the use of extrinsic dexterity relies on three kinds of external forces: gravitational forces, inertia induced forces, and external forces due to contacts.

First, gravitational forces have been used to reorient an object. In [7], an object is placed on a plate on which it can slip. After tilting the plate with a given sequence of motions, it is possible to determine the orientation of the object, achieving sensorless manipulation. In [8], the use of two end-effectors in synergy with gravity allows more complex manipulation tasks of an object resting on these end-effectors. Reorientation inside a gripper has also been studied in [9]. In this work, an algorithm capable of controlling the orientation of an object pinched by two fingers using only gravity has been developed. To do so, the slip between the fingers and the object is controlled by modifying the distance between the fingers. Further studies on the needed forces exerted by the gripper have also been conducted in [10].

Second, similar rotations can be achieved using the second kind of forces, *i.e.*, forces due to inertia. In [11], an open-loop control in three phases has been implemented. In the first phase, the gripper and the object are moved with the same velocity. In the second phase, the gripper stops and the distance between the fingers is increased. In the third phase, the object rotates due to its inertia. The friction parameters are determined experimentally using a *Q-learning* algorithm. In [12], a closed-loop control is proposed. In [13], inertial forces are also used to make an object slip thanks to the acceleration of the gripper.

Third, the use of external contacts has been considered, such as in [14]. In this article, a grasped cylinder is manipulated by pushing it against a fixed surface. Interesting work has also been proposed in [15] where an object grasped by a robotic gripper is manipulated thanks to a fixed part. This method has been extended in [16] to manage more complex contacts between the object, the gripper and the fixed part. A motion planning algorithm based on the motion cone principle is developed. The motion cone principle was introduced by Mason in [17] and represents the set of possible movements of an object pushed with friction. Another example of reorientation using a fixed surface can be found in [18] where two primitives are introduced: the *pivot-on-support* and the *roll-on-support*. A formulation using a Second-Order Cone Program (SOCP) is presented in [19], which is an interesting avenue since it represents an approach based on convex optimization.

Finally, different kinds of external forces can be used in a coordinated fashion such as in [20] where gravity, inertia, and contacts are used. The authors provide interesting insight about manipulation with extrinsic dexterity and develop twelve regrasping sequences. An extensive experimental evaluation is also carried out with 1200 trials and the construction of a set of manipulation tasks is performed using a *grasp graph*. We can also cite the work in [21], where gravity is used to move an object while the reorientation is achieved by a succession of external contacts even if the initial orientation is unknown, providing sensorless manipulation.

As we have seen, groundwork has been presented in the literature for the use of extrinsic dexterity. However, the state of the art still lacks a generic mathematical model for object manipulation using external resources. This paper fills this gap and is structured as follows. First, a mathematical model for the in-hand manipulation of an object exploiting extrinsic dexterity is developed and exploited for several case studies in Section 2. Then, the introduction of an assistive tool to aid the regrasping process is studied in Section 3. Finally, conclusions are drawn and future avenues for this work are proposed in Section 4.

2. MODELLING AND IMPLEMENTATION

The model developed in this work is described here. After that, the model is applied to two simple one-dimensional problems for illustration purposes.

2.1. Development of the model

The aim of this model is to obtain the equations of motion of the object in the gripper. Knowing that the gripper can move, the model is analogous to the dynamics of a rigid body in a non-inertial frame. More specifically, we need to determine the displacement of the object $\mathcal{C}$ with respect to the gripper $\mathcal{B}$ using a fixed frame $\mathcal{A}$. Using the composition law of SE(3), the relative body velocity can be express as

$$\mathbf{V}_{\mathcal{C}}^{\mathcal{A}} = \left[\mathrm{Ad}_{\mathbf{R}_{\mathcal{B}}^{\mathcal{A}}\mathbf{p}_{\mathcal{C}}^{\mathcal{B}}}\right]\mathbf{V}_{\mathcal{B}}^{\mathcal{A}} + \left[\mathrm{Ad}_{\mathbf{R}_{\mathcal{B}}^{\mathcal{A}}}\right]\mathbf{V}_{\mathcal{C}}^{\mathcal{B}}\,, \tag{3}$$

where $\mathbf{V}_{\mathcal{C}}^{\mathcal{A}}, \mathbf{V}_{\mathcal{B}}^{\mathcal{A}}, \mathbf{V}_{\mathcal{C}}^{\mathcal{B}} \in \mathbb{R}^6$ are the generalized relative velocities, $\left[\mathrm{Ad}_{\mathbf{R}_{\mathcal{B}}^{\mathcal{A}}\mathbf{p}_{\mathcal{C}}^{\mathcal{B}}}\right]$ and $\left[\mathrm{Ad}_{\mathbf{R}_{\mathcal{B}}^{\mathcal{A}}}\right]$ are respectively the adjoint representation of the relative translation and rotation, written as

$$\left[\mathrm{Ad}_{\mathbf{R}_{\mathcal{B}}^{\mathcal{A}}\mathbf{p}_{\mathcal{C}}^{\mathcal{B}}}\right] = \begin{bmatrix}\mathbf{1}_{3\times3} & -\left[\mathbf{R}_{\mathcal{B}}^{\mathcal{A}}\mathbf{p}_{\mathcal{C}}^{\mathcal{B}}\right]_{\times} \\ \mathbf{0}_{3\times3} & \mathbf{1}_{3\times3}\end{bmatrix} \quad \text{and} \quad \left[\mathrm{Ad}_{\mathbf{R}_{\mathcal{B}}^{\mathcal{A}}}\right] = \begin{bmatrix}\mathbf{R}_{\mathcal{B}}^{\mathcal{A}} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{R}_{\mathcal{B}}^{\mathcal{A}}\end{bmatrix}$$

with $\mathbf{R}_{\mathcal{B}}^{\mathcal{A}} \in \mathrm{SO}(3)$ the rotation matrix between the frame attached to the gripper $\mathcal{B}$ and the fixed frame $\mathcal{A}$, $\mathbf{p}_{\mathcal{C}}^{\mathcal{B}} \in \mathbb{R}^3$ the position of the object $\mathcal{C}$ relative to the gripper $\mathcal{B}$, and $\left[\mathbf{R}_{\mathcal{B}}^{\mathcal{A}}\mathbf{p}_{\mathcal{C}}^{\mathcal{B}}\right]_{\times}$ is the skew-symmetric cross product matrix of the vector $\mathbf{R}_{\mathcal{B}}^{\mathcal{A}}\mathbf{p}_{\mathcal{C}}^{\mathcal{B}}$. Using the Newton-Euler equations, the motion of $\mathcal{C}$ with respect to $\mathcal{A}$ is

$$\mathbf{F}^{\mathcal{A}} = \mathbf{M}\dot{\mathbf{V}}_{\mathcal{C}}^{\mathcal{A}} + \mathbf{F}_{\text{inertial}}^{\mathcal{A}}\,, \tag{4}$$

where $\mathbf{F}^{\mathcal{A}} \in \mathbb{R}^6$ is the wrench applied to the object, $\mathbf{M} \in \mathbb{R}^{6\times6}$ is the generalized inertia matrix of the object, and $\mathbf{F}_{\text{inertial}}^{\mathcal{A}} \in \mathbb{R}^6$ is the pseudo-wrench created by the acceleration of the gripper. Substituting the total time derivative of Eq. (3) in Eq. (4) and isolating the acceleration of the object relative to the gripper, we get the final equation of the model, therefore

$$\dot{\mathbf{V}}_{\mathcal{C}}^{\mathcal{B}} = \left[\mathrm{Ad}_{\mathbf{R}_{\mathcal{B}}^{\mathcal{A}}}\right]\left(\mathbf{M}^{-1}\left(\mathbf{F}_{\text{gravity}}^{\mathcal{A}} + \mathbf{F}_{\text{external}}^{\mathcal{A}} + \mathbf{F}_{\text{friction}}^{\mathcal{A}}\right) - \mathbf{F}_{\text{inertial}}^{\mathcal{A}}\right)\,. \tag{5}$$

The wrench $\mathbf{F}^{\mathcal{A}}$ as been separated in three terms to better represent the possible kinds of external forces use by extrinsic dexterity. The first one $\mathbf{F}_{\text{gravity}}^{\mathcal{A}}$ represents the gravitational force acting on the object and is trivial to compute. The second one $\mathbf{F}_{\text{external}}^{\mathcal{A}}$ is the contact force between the object and an external body, it can be computed in different ways. The approach used in the implementation of this paper is a simple inequality to bound the motion of the object in space. Finally, the third one $\mathbf{F}_{\text{friction}}^{\mathcal{A}}$ is the friction between the object and the gripper. If the gripper is a parallel-plate gripper, the friction force is then restrained to the plane of the plates. This wrench is computed using the maximum dissipation principle introduced in [22]. According to this principle, the contact forces and moment that generate the highest power dissipation within the friction ellipsoid are chosen from all the possible options, described mathematically by

$$\begin{aligned} &\max \quad -\mathbf{V}_{\mathcal{C}}^{\mathcal{B}} \cdot \mathbf{F}_{\text{friction}}^{\mathcal{A}} \\ &\text{s.t.} \quad \left(\mathbf{V}_{\mathcal{C}}^{\mathcal{B}} \oslash \mathbf{F}_{\text{friction}}^{\mathcal{A}}\right) \cdot \left(\mathbf{V}_{\mathcal{C}}^{\mathcal{B}} \oslash \mathbf{F}_{\text{friction}}^{\mathcal{A}}\right) - \mu^2\lambda_n^2 \leq 0\,, \end{aligned} \tag{6}$$

where $\mathbf{F}_{\text{friction}}^{\mathcal{A}}$ contains the optimization variables, μ is the coefficient of friction, λ_n is the normal contact force (*i.e.*, grip force in our case) and $\oslash$ is the Hadamard division [23]. Using the Fritz-John optimality conditions and the fact that, for planar sliding, the friction wrench has to be at the boundary of the friction ellipsoid, the optimization procedure Eq. (6) can be reduced to the solution of a system of four quadratic equations, see [24] for more details.

To be able to carry out simulations, the model must be discretized. For each time step, the object speed is computed thanks to the acceleration (Eq. (5)) and the speed at the previous step. Moreover, the aforementioned four-equation system must be solved at each time step to compute the friction wrench.

2.2. Translational one-dimensional problem

The model developed in Eq. (5) is firstly exploited to study the one-dimensional problem of a translation, illustrated in Fig. 1. In this figure, g is the gravitational acceleration vector, x_1 the displacement of the gripper (*i.e.*, $\mathcal{B}$ in respect to $\mathcal{A}$), x_2 the displacement of the object with respect to the gripper (*i.e.*, $\mathcal{C}$ in respect to $\mathcal{B}$), $F_{friction}$ the friction force applied by the gripper, $F_{external}$ the external force induced by the contact with the mechanical stop, and $F_{gravity}$ the gravitational force. Two case studies are proposed. In the first one, only the inertia of the object is considered, while a mechanical stop to generate external contacts is used in the second one.

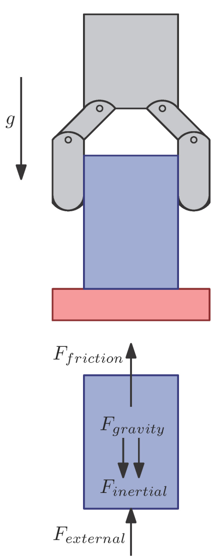


Fig. 1. Translational one-dimensional problem: the blue rectangle is the object and the red one a mechanical stop.

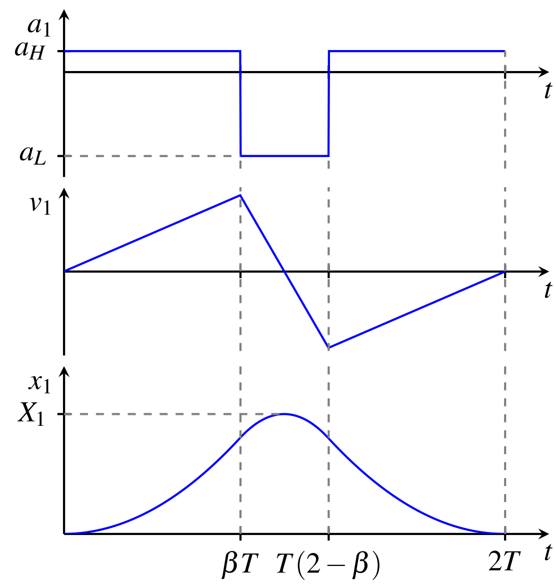


Fig. 2. Motion profile of the gripper to use inertial force to manipulate an object.

2.2.1. Use of inertial forces

When inertial forces are considered, the acceleration profile of the gripper is important. Therefore, highly dynamic motions are generated by adjusting the acceleration direction to move the object up or down with respect to the gripper, as already proposed in [25]. To obtain a general formulation of the motion, we introduce the ratio β that represents the selected ratio between the acceleration and deceleration. The acceleration is then written as

$$a_1 \equiv \frac{d^2x_1}{dt^2} = a_H H(t) + (a_L - a_H)H(t - \beta T) + (a_H - a_L)H(t - (2-\beta)T) \,, \tag{7}$$

with $H(\cdot)$ the Heaviside step function, T the displacement time, a_H and a_L the acceleration and the deceleration respectively. This motion profile is visually represented at Fig. 2. The displacement amplitude of the gripper is noted X_1, and can be expressed as

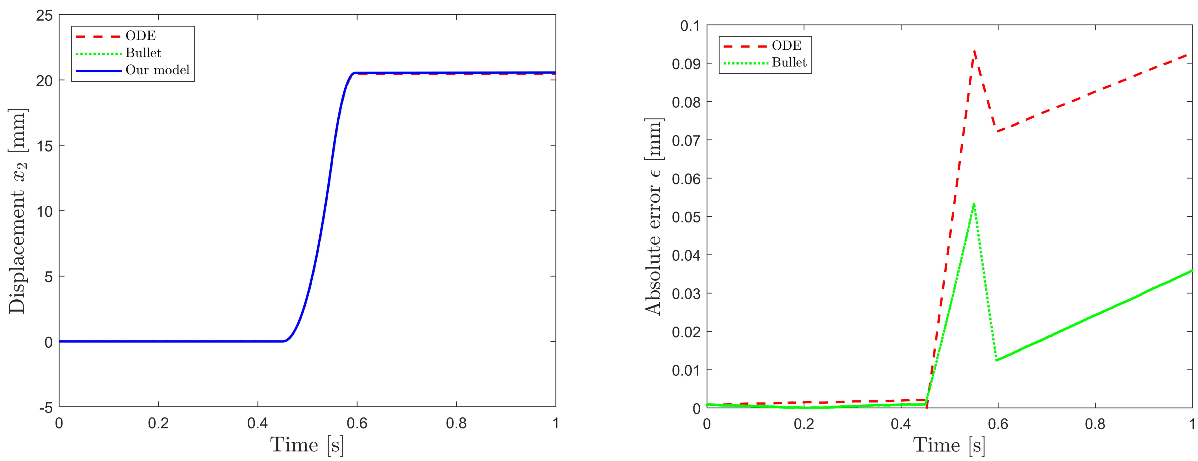
$$X_1 = \frac{1}{2} a_H \beta^2 T^2 \left(1 - \frac{a_H}{a_L}\right) . \tag{8}$$

Thus, the accelerations needed to produce a given displacement with a given ratio β can be obtained as

$$a_H = \frac{2X_1}{\beta T^2} , \tag{9a}$$

$$a_L = \frac{2X_1}{(\beta - 1)T^2} . \tag{9b}$$

Fig. 3 shows a 20 mm relative displacement of the object with respect to the gripper for $\beta = 0.9$, $X_1 = 0.1$ m, $T = 0.5$ s, $m = 0.1$ kg and $\mu = 0.5$. To validate these results, the displacement is computed with the model developed in Section 2.1 using *MATLAB* and compared to simulations carried out using the *CoppeliaSim* software with two different physics engines: *ODE* and *Bullet*. All three curves superimpose, validating the proposed model for this case study. Furthermore, an unwanted effect is observed in the simulations carried out by the two physics engines. Indeed, after the relative displacement, the object is not completely stopped by the friction with the gripper. This highlights the difficulty of modeling dry friction. However, our model is capable of eliminating this undesirable drift and thus performs better.

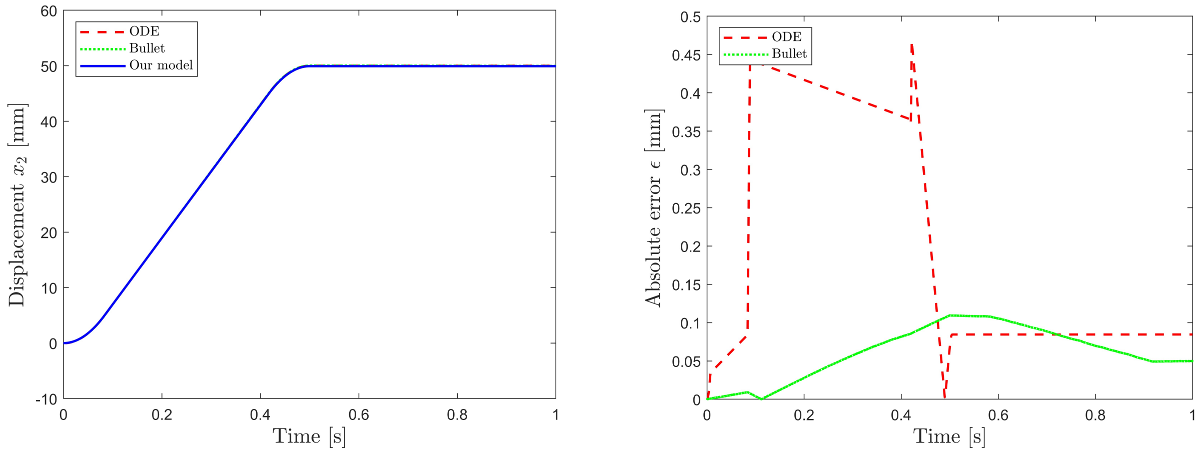


(a) (b)

Fig. 3. Model-based computation and simulations of the displacement x_2 of the object with respect to the gripper using inertial forces for $\beta = 0.9$, $X_1 = 0.1$ m, $T = 0.5$ s, $m = 0.1$ kg and $\mu = 0.5$. (a) displacement obtained with the three simulations, (b) absolute error between the two physics engines and our model.

2.2.2. Use of contact forces

When contact forces are considered, a contribution from the acceleration of the gripper is not needed. Thus, a trapezoidal velocity profile is implemented to position the object in the gripper. For a maximum velocity $V_{max} = 0.12$ m/s, a displacement $X_1 = 0.05$ m, a duration of the displacement $T = 0.5$ s, $m = 0.1$ kg, and $\mu = 0.5$, we obtain the displacement depicted in Fig. 4. Contact is made at $t = 0$ s. A displacement of 50 mm in the direction opposite to gravity is obtained as desired. Both simulations using *CoppeliaSim* are also carried out to validate the model. All three curves superimpose, validating the equations for the case of external contact forces. The small difference between the three simulations is caused by the different contact models used, which is elastic for the two physics engines.



(a) (b)

Fig. 4. Model-based computation and simulations of the displacement x_2 of the object with respect to the gripper using contact forces for $V_{max} = 0.12$ m/s, $X_1 = 0.05$ m, $T = 0.5$ s, $m = 0.1$ kg, and $\mu = 0.5$. (a) displacement obtained with the three simulations, (b) absolute error between the two physics engines and our model.

2.3. Rotational one-dimensional problem

In this section, one-dimensional rotations are considered. The chosen case study is illustrated in Fig. 5. In this figure, g is the gravitational acceleration vector, θ_1 the rotation of the gripper (*i.e.*, $\mathcal{B}$ in respect to $\mathcal{A}$), θ_2 the rotation of the object with respect to the gripper (*i.e.*, $\mathcal{C}$ in respect to $\mathcal{B}$), $F_{friction}$ the friction torque applied by the gripper, and $F_{gravity}$ the gravitational force. In this case study, the object does not undergo any translation but a one-DoF rotation around a fixed contact point between the gripper and the object. A constant pressure distribution is considered between the gripper and the object. The rotation of the object is taking place around the pressure centre depicted in orange, which is located in the centroid of the contact surface. For this study, we assume that the surface pressure does not vary and that the pressure centre is fixed during the whole displacement.

In this preliminary work, we only study the use of inertial forces to move the object with respect to the gripper. Similarly to the translational problem using inertial forces, we need displacements of the gripper with high dynamics. Therefore, we exploit the same acceleration profile, proposed in Eq. (7), but this time as an angular acceleration. The angular displacements of the object with respect to the gripper obtained with this model and with each of the simulations using *CoppeliaSim* are depicted in Fig. 6.

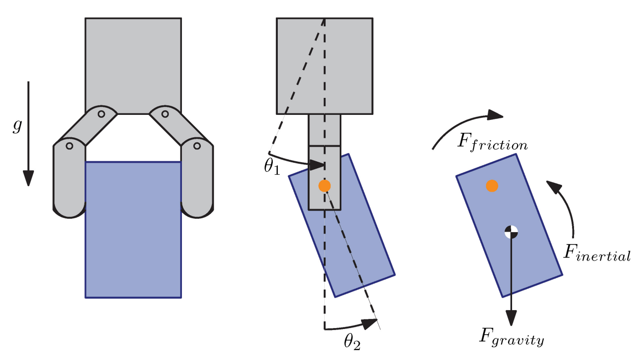


Fig. 5. Rotational one-dimensional problem: the blue rectangle is the object.

A relative displacement of roughly 140° is obtained with our model, while smaller values are obtained in the other simulations. This difference is due to the approximations used by the physics engines to simulate the contact surfaces. Indeed, with these tools, the contact surfaces are modelled using multiple contact points at the vertices of the surface [26, 27]. Alternatively, with the proposed mathematical model, the whole surface is modelled. Moreover, the *Bullet* physics engine uses a linear approximation of the friction cone, which explains the smaller displacements observed. It is conjectured that the model presented here should be closer to the real world than the two physics engines for this problem. However, this conjecture must be confirmed through experimental evaluations.

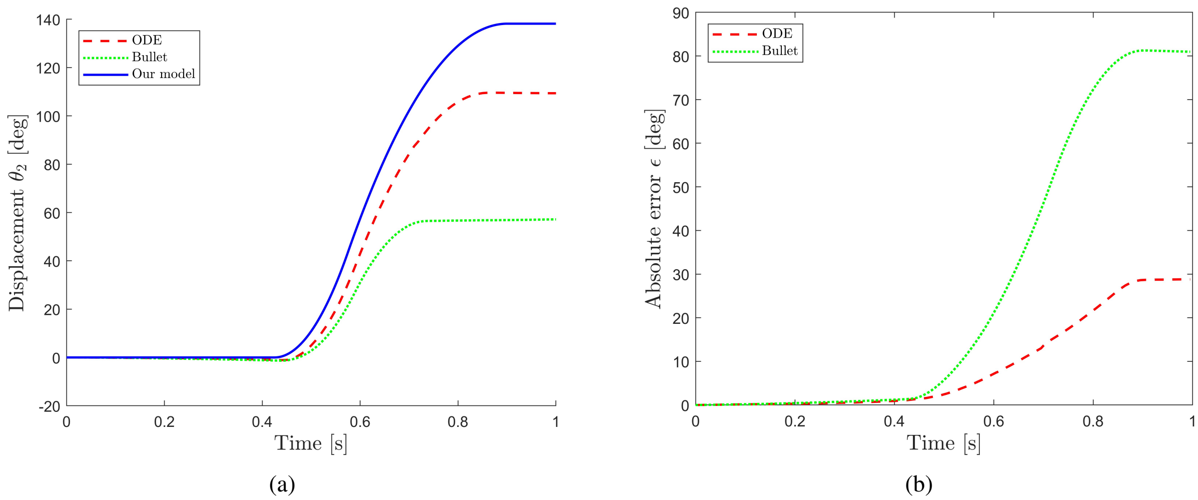


Fig. 6. Model-based computation and simulations of the displacement θ_2 of the object with respect to the gripper for $\beta = 0.7$, $\theta_1 = 100°$, $T = 3$ s, $m = 0.1$ kg, $\boldsymbol{I} = \text{diag}(0.5, 0.5, 0.04)$ kg.m^2 and $\mu = 0.5$. (a) displacement obtained with the three simulations, (b) absolute error between the two engines and our model.

These three simple case studies showed the relevance of the proposed mathematical model to achieve in-hand motions using extrinsic dexterity. In addition to introducing the new model and its evaluation in multiple case studies, this work is also a first step towards the implementation of such object manipulations with the development of dedicated tools.

3. DESIGN OF AN ASSISTIVE TOOL FOR IN-HAND MANIPULATION

In the manipulation and grasping of objects, it is common for humans to use both hands, for instance for tight packaging applications. In some situations, the use of two hands simplifies the manipulation tasks, even though the role of the second hand may be relatively simple (*e.g.*, stabilizing the package or pushing against objects). This observation led to the concept of an assistive tool, which involves a combination of external environmental features.

3.1. A first manipulation task

To illustrate the design methodology of an assistive tool, a simple manipulation task is analyzed. The task consists in the manipulation of a unit square using two infinite flat surfaces. These surfaces are tilted at an angle θ and the unit square is pushed against the assistive tool with an angle of attack ϕ (see Fig. 7).

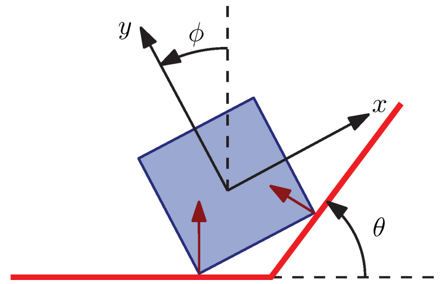


Fig. 7. Manipulation of a unit square at an angle of attack ϕ using two infinite flat surfaces forming an angle θ.

First, consider two contact points without any contribution from friction and gravitational forces (manipulation on a horizontal surface). Each contact point generates a unilateral constraint on the twist of the unit square. These constraints may be represented as inequalities that the twist of the unit square needs to satisfy. For the first contact, the constraint on the 'left' plane takes the form

$$V_x \sin\phi + V_y \cos\phi - \frac{1}{2}(1-\sin\phi)\omega_z \geq 0\,. \tag{10}$$

The second constraint, induced by the contact with the 'right' tilted plane is given by

$$-V_x \sin(\theta-\phi) + V_y \cos(\theta-\phi) - \frac{1}{2}(1-\sin(\theta-\phi))\omega_z \geq 0\,, \tag{11}$$

where V_x, V_y and ω_z are respectively the velocity and the angular velocity in the xy plane. The feasible twist set is represented by a cone rooted at the origin, referred to as a homogeneous polyhedral convex cone [28] (see Fig. 8). In an ideal scenario, the object is completely fixed before manipulation. Then, the gripper can move freely on the object and any manipulation tasks can be easily accomplished thereafter. This scenario is only possible with a specific fixture for a single object's geometry. Indeed, this technique is not valid when multiple objects need to be manipulated, although it is possible to determine the degree of freedom of the object from the polyhedral convex cone created by the contact points. This is because the region inside this cone represents all possible twists of the object. By minimizing the volume of this cone for several objects to be manipulated, an appropriate assistance tool can be identified for many objects.

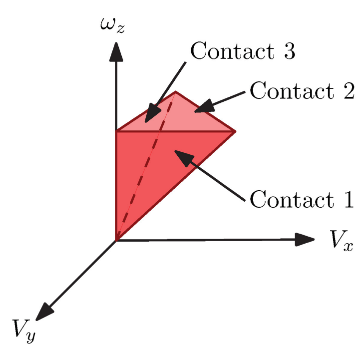


Fig. 8. Example of a polyhedral convex cone representing the feasible twist set of a constrained object. For illustration purpose, this is not the resulting cone from the unit square manipulation task.

In the manipulation task presented in Fig. 7, the variety of objects to be manipulated can be represented by the angle of attack of the unit square ϕ and the optimization of the assistive tool provides the angle θ.

Thus, the optimization problem is written as follows

$$\theta^* = \underset{\theta}{\operatorname{argmin}} \sum_{\phi} V(\phi, \theta), \tag{12}$$

where $V(\phi, \theta)$ is the volume of the polyhedral convex cone, which is a function of the two angles. This volume can be computed by discretizing the space bounded by a cuboid (Fig. 9). To simplify the analysis without loss of generality, the volume of the cuboid is represented by the maximum volume V_{max}, which is set to unity

$$V_{max} = \Delta V_x \cdot \Delta V_y \cdot \Delta \omega_z = 1. \tag{13}$$

The aspect ratio of the cuboid depends strongly on the size of the object to be manipulated. In this particular case (for a unit square), it can be observed that a coefficient $\frac{1}{2}$ appears in front of ω_z (see Eq. (10) and Eq. (11)). Therefore, a logical choice consists in taking $\Delta\omega_z = 2$ and $\Delta V_x = \Delta V_y = \frac{1}{\sqrt{2}}$. In general, a continuous variation of the cuboid dimensions is performed to assess the invariance behaviour of the optimization problem stated in Eq. (12). The optimization procedure for the case presented here gives an optimal angle of $\theta^* = 90°$. This result is not surprising, as it restricts the unit square to an octant of the velocity coordinate system.

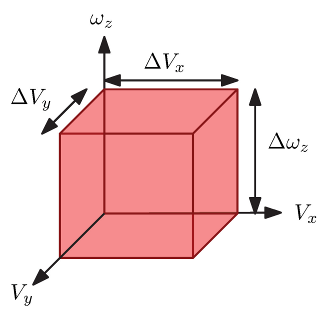


Fig. 9. Cuboid bounding the twist space.

3.2. Impact of friction

Adding friction at the contact points between the object and the assistive tool introduces further restraints on the object. From the constraint equations, each contact point generates two friction forces, one at each boundary of the friction cone (Fig. 10). Analyzing these interactions without friction is thus a conservative assumption, since the use of friction forces in the optimization of the geometry of the assistive tool complicates the analysis. Thus, a purely geometric analysis is then not possible and a force analysis is needed to improve the model.

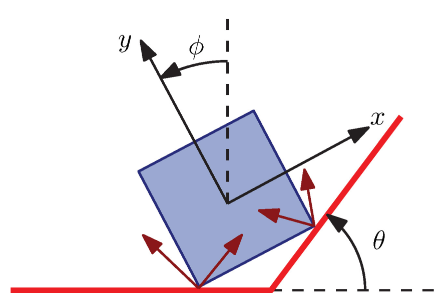


Fig. 10. Manipulation of a unit square at an angle ϕ using two infinite flat surfaces tilted of an angle θ with frictional contacts.

4. CONCLUSION AND FUTURE WORK

In this work, a new mathematical model was developed to generalize the in-hand manipulation of an object using extrinsic dexterity. To evaluate the accuracy of this model, comparisons with two physics engines have been carried out. It was then shown that the new model allows to solve multiple manipulation problems and is promising for further developments. After that, preliminary studies on the design of an assistive tool to aid manipulation tasks have been carried out and led to promising avenues to further the research.

Although rather preliminary, the study presented in this paper and the simulation results are paving the way to a systematic investigation of the use of extrinsic dexterity for in-hand manipulation. The proposed next tasks involve a more efficient way to compute the volume of the polyhedral convex cone. The goal is to develop an optimized model by introducing more parameters in the process (number of objects and of assistive tool geometries). Also, recent investigations have demonstrated that a better way to constrain the object may be to use an adaptable assistive tool. For example, an architecture inspired from an underactuated finger equipped with passive springs [29].

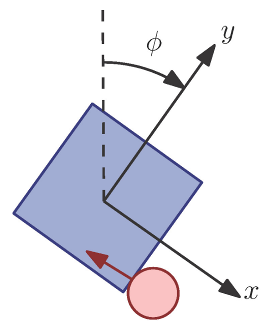


Fig. 11. Rotation of an object using a simple rod.

So far, the optimization is based on the case for which the object is fully constrained during manipulation. However, it is thought that certain manipulation tasks do not require the object to be fully constrained by the assistive tool. For example, a rotation of the object in the gripper can be achieved with a very simple assistive tool like the one represented in Fig. 11. Further investigations may provide the necessary insight to help define a more suitable assistive tool model by dividing the manipulation tasks into multiple elementary tasks, referred to as primitives, (rotation, translation, *etc*.). The optimization of all geometries may each utilize a model from these specific primitives. By combining such geometries, an innovative design to define an efficient assistive tool may be envisioned.

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