INFLUENCE OF DESIGN PARAMETERS ON THE SINGULARITIES AND WORKSPACE OF A 3-RPS PARALLEL ROBOT

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ABSTRACT

This paper presents the variations in the workspace, singularities and joint space with respect to the design parameter *k* of the 3-RPS parallel manipulator. Also the influence on the parasitic motions due to the design parameters is studied, which plays an important role in the selection of the manipulator for a desired task. The cylindrical algebraic decomposition method and Gröbner based computations are used to model the workspace and joint space with the parallel singularities in 2R1T and 3T projection spaces, where the orientation of the mobile platform is represented by using quaternions. These computations are useful to select the optimum value for the design parameter such that the parasitic motions can be limited to specific values. Depending on the design parameter *k*, three different configurations of the 3-RPS parallel robot are analyzed.

Keywords: Parallel Robot; Singularities; 3-RPS; Workspace; Kinematics.

INFLUENCE DES PARAMÈTRES DE CONCEPTION SUR LES SINGULARITÉS ET L'ESPACE DE TRAVAIL D'UN 3-RPS ROBOT PARALLÈLE

RÉSUMÉ

Cet article présente les variations de l'espace de travail, des singularités et de l'espace articulaire en fonction d'un paramètre de conception *k* qui est le ratio entre les dimensions de la plate-forme mobile et les dimensions de la base du manipulateur parallèle 3-RPS. On étudie également l'influence de ce paramètre de conception sur les mouvements parasites, qui joue un rôle important dans la sélection du robot pour une tâche souhaitée. La méthode de décomposition algébrique cylindrique et les bases de Gröbner sont utilis'es pour modéliser l'espace de travail et l'espace articulaire ainsi que les singularités parallèles dans les espaces de projection 2R1T and 3T, où l'orientation de la plate-forme mobile est représentée en utilisant des quaternions. Ces calculs sont utiles pour sélectionner la valeur optimale pour le paramètre de conception de telle sorte que les mouvements parasites puissent être limités à des valeurs spécifiques. En fonction du paramètre *k*, trois configurations différentes du robot parallèle 3-RPS sont analysées.

Mots-clés : Robot parallèle ; Singularité ; 3-RPS ; Espace de travail ; Cinématique.

1. INTRODUCTION

The study of workspace, joint space and singularities together assists the engineers and researchers in the efficient task planning and the selection of the particular configuration of the manipulator for a desired task. The accurate computation of the workspace and joint space for 3-RPS parallel robotic manipulator is a highly addressed research work across the world. It is well a known feature that this robot admits two operation modes. As there exists parasitic motions for 3RPS parallel robot, a complete workspace analysis of this mechanism is possible, only when the analysis is done in both 2R1T and projection spaces for both the operation modes.

For the parallel robots with several inverse and direct kinematic solutions, the aspects are defined as the maximal singularity-free sets in the workspace or the cross-product of the joint space by the workspace. An assembly mode is associated with a solution for the Direct Kinematic Problem (DKP) and a working mode for the Inverse Kinematic Problem (IKP). The notion of aspect, previously defined in [1] for serial robots and in [2] for parallel robot with one operation mode, and in [3, 4] extended for a parallel robot with several operation modes. There are different techniques based on geometric [5, 6], discretization [7–9], and algebraic methods [4, 10–13] which can be used to compute the workspace of parallel robot. Practically, a change of assembly mode may occur during the execution of a trajectory between two configurations in the workspace which is shown in [4, 14] for the 3-RPS parallel robot with similar base and mobile platforms.

This paper presents the results which are obtained by applying algebraic methods for the workspace, singularities and joint space analysis of the reconfigurable 3RPS parallel robot with changeable size ratio of moving and base platform. The CAD algorithm is used to study the workspace and joint space, and a Gröbner based elimination method is used to compute the parallel singularities. The outline of this paper is as follows. Section 2 describes the architecture and kinematic equations of the manipulator. Section 3 discusses the influence of design parameter on parallel singularities. Section 4 and 5 presents the workspace and joint space analysis of the manipulator. Section 6 finally concludes the paper.

2. MECHANISM UNDER STUDY

The robot under study is the 3-RPS parallel robot with three degrees of freedom and has been studied by many researchers [15–18]. It is the assembly of two equilateral triangles (the base and the moving platform) by three identical RPS legs where R is a revolute passive joint, P an actuated prismatic joint and S a passive spherical joint. Thus, the revolute joint is connected to the fixed base and the spherical joint to the mobile platform.

Considering the 3-RPS parallel manipulator, as shown in Fig. 1, the fixed base consists of an equilateral triangle with vertices A_1, A_2 and A_3, and circumradius g. The moving platform is another equilateral triangle with vertices B_1, B_2 and B_3 and varying circumradius h, and circumcenter P. The two design parameters g and h are positive numbers. Connecting each of the vertices' pairs A_i, B_i $(i = 1,2,3)$ by a limb, a rotational joint lies at A_i and a spherical joint lies at B_i. ρ_i denotes the length of each limb and the motion control is done through an actuated prismatic joint. Thus, there are five parameters, namely g, h, ρ_1, ρ_2 and ρ_3. The g and h parameters ($h/g = k$) determine the design of the manipulator (shown in fig. 2) whereas the joint parameters ρ_1, ρ_2 and ρ_3 determine the motion of the robot. Spatial rotations in three dimensions can be parametrized using Euler angles [19], unit quaternions [20] or dual quaternions [15]. The quaternion representation is used for modeling the orientation as quaternions do not suffer from singularities as Euler angles do. Moreover, to transform the trigonometric equations to algebraic equations, we may either introduce the singularity of the transformation $t = \tan(\alpha/2)$ or replace the angle α by two parameters $\cos_\alpha$ and $\sin_\alpha$ with $\cos_\alpha^2 + \sin_\alpha^2 - 1$. In

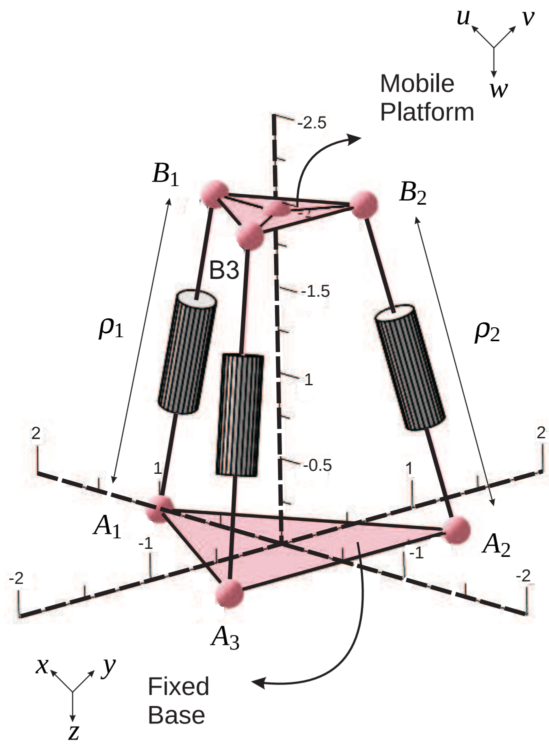


Fig. 1. 3-R<u>P</u>S parallel robot

addition, it is easier to represent workspace sections with the quaternions than the dual quaternions because the position is simply defined the Cartesian coordinates.

A quaternion **q** is defined by

$$\mathbf{q} = q_1 + q_2\mathbf{i} + q_3\mathbf{j} + q_4\mathbf{k} \tag{1}$$

The quaternion rotation matrix is then

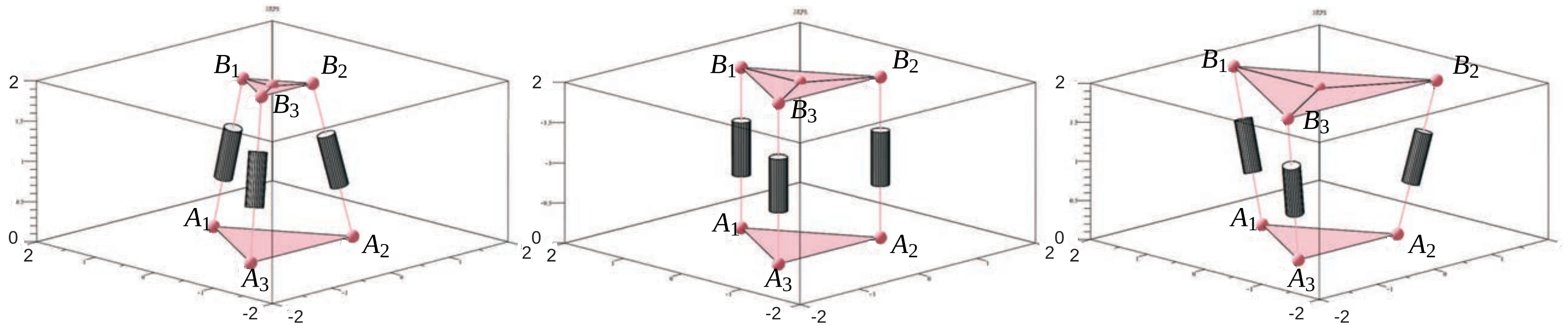
$$\mathbf{Q} = \begin{bmatrix} 2{q_1}^2 + 2{q_2}^2 - 1 & -2q_1q_4 + 2q_2q_3 & 2q_1q_3 + 2q_2q_4 \\ 2q_1q_4 + 2q_2q_3 & 2{q_1}^2 + 2{q_3}^2 - 1 & -2q_1q_2 + 2q_3q_4 \\ -2q_1q_3 + 2q_2q_4 & 2q_1q_2 + 2q_3q_4 & 2{q_1}^2 + 2{q_4}^2 - 1 \end{bmatrix} \tag{2}$$

with $q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$. The transformation from the moving frame to the fixed frame can be described by a position vector OP and a 3×3 rotation matrix **R**. Let **u**, **v** and **w** be the three unit vectors defined along the axes of the moving frame, then the rotation matrix can be expressed in terms of the coordinates of **u**, **v** and **w** as:

$$\mathbf{R} = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix} \tag{3}$$

The vertices of the base triangle and mobile platform triangle are

$$\mathbf{A}_1 = \begin{bmatrix} g \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{A}_2 = \begin{bmatrix} -g/2 \\ g\sqrt{3}/2 \\ 0 \end{bmatrix} \quad \mathbf{A}_3 = \begin{bmatrix} -g/2 \\ -g\sqrt{3}/2 \\ 0 \end{bmatrix} \tag{4}$$



k = 0.5 (a) k = 1 (b) k = 1.5(c)

Fig. 2. Virtual model of 3-R<u>P</u>S parallel manipulator with different design parameter k, k = 0.5 (a), k = 1 (b), k = 1.5 (c)

$$\mathbf{b}_1 = \begin{bmatrix} h \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} -h/2 \\ h\sqrt{3}/2 \\ 0 \end{bmatrix} \quad \mathbf{b}_3 = \begin{bmatrix} -h/2 \\ -h\sqrt{3}/2 \\ 0 \end{bmatrix} \tag{5}$$

The coordinates of $\mathbf{b}_i$ with respect to fixed frame reference are obtained by $\mathbf{B}_i = \mathbf{P} + \mathbf{R}\mathbf{b}_i$ for $i = 1,2,3$. Also the coordinates of the center of the mobile platform in the fixed reference is $\mathbf{P} = [x\ y\ z]^T$. The distance constraints yields:

$$||\mathbf{A}_i - \mathbf{B}_i|| = \rho_i^2 \quad \text{for} \quad i = 1,2,3 \tag{6}$$

As A_i are revolute joints, the motion of the B_i are constrained in planes. This leads to the three constraint equations:

$$u_y h + y = 0 \tag{7}$$
$$y - u_y h/2 + \sqrt{3} v_y h/2 + \sqrt{3}x - \sqrt{3} u_x h/2 + 3 v_x h/2 = 0 \tag{8}$$
$$y - u_y h/2 - \sqrt{3} v_y h/2 - \sqrt{3}x + \sqrt{3} u_x h/2 + 3 v_x h/2 = 0 \tag{9}$$

Solving with respect to x and y we get:

$$y = -h u_y \tag{10}$$
$$x = h\left(\sqrt{3}u_x - \sqrt{3}v_y - 3u_y + 3v_x\right)\sqrt{3}/6 \tag{11}$$

In Eqs. (6, 8, 9), we substitute x,y using Eqs. (10) and (11), and $\mathbf{u}, \mathbf{v}, \mathbf{w}$ by quaternion expressions using Eq. (2). Then Eqs. (8) and (9) become $q_1 q_4 = 0$. Thus, we have either $q_1 = 0$ or $q_4 = 0$. This property is associated with the notion of operation mode. Let OM_1 be the operation mode for $q_1 = 0$ and OM_2 for $q_4 = 0$. To obtain the algebraic equations, we replace $\sqrt{3}$ by the variable s_3 and add the equation $s_3^2 - 3 = 0$ and the constraint $s_3 > 0$.

3. INFLUENCE OF DESIGN PARAMETER ON PARALLEL SINGULARITIES

Differentiating the constraints equations with respect to time leads to the velocity model:

$$\mathbf{A}\mathbf{t} + \mathbf{B}\dot{\mathbf{q}} = 0 \tag{12}$$

where $\mathbf{A}$ and $\mathbf{B}$ are the parallel and serial Jacobian matrices, respectively, $\mathbf{t}$ is the velocity of P and $\dot{\mathbf{q}}$ the joint velocities. The parallel singularities occur whenever $\det(\mathbf{A}) = 0$.

As there exist parasitic motions for 3R$\underline{\text{P}}$S parallel robot, it is possible to analyze the singularities in two different projection spaces, $[p_z\ q_2\ q_3]$ and $[p_x\ p_y\ p_z]$. Figures 3 and 4 represent the projection of parallel singularities in projection space $[p_x\ p_y\ p_z]$ for OM^1 and OM^2, respectively. The variations in the parallel singularities surface for different design parameter k are shown in Figures 3 and 4. Also the top view of these singularity surfaces is presented to analyze the change in the singularities surface boundaries with the change in the value of k. As the value of k increases the connectivity in the workspace region decreases, can be seen in Figures 3(e-h) and 4(e-h).

4. WORKSPACE ANALYSIS OF THE 3-R$\underline{\text{P}}$S PARALLEL ROBOT

The 3-R$\underline{\text{P}}$S parallel robot is three degrees of freedom parallel robot with two rotational [2R] and one translational [1T] motions, but still there exist two other translational motions in the x and y direction. These motions are termed as parasitic motions as they depend on other existing rotational motions. A complete

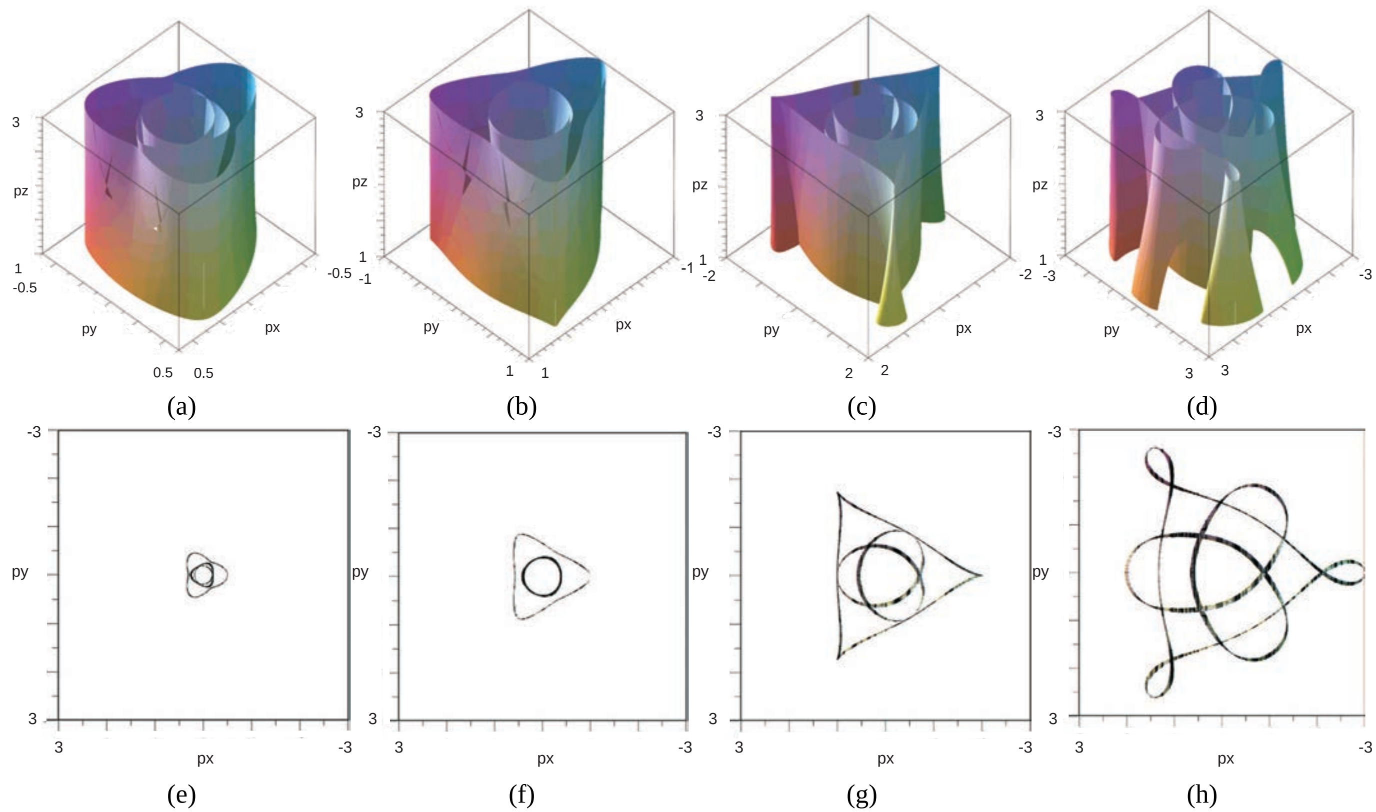


Fig. 3. Projection of parallel singularities in projection space $[p_x\ p_y\ p_z]$ for OM^1. Variation in the parallel singularities surface for different design parameter k = 0.5 (a), k = 1 (b), k = 2 (c) and k = 3 (d), top view of the parallel singularities surface, k = 0.5 (e), k = 1 (f), k = 2 (g) and k = 3 (h).

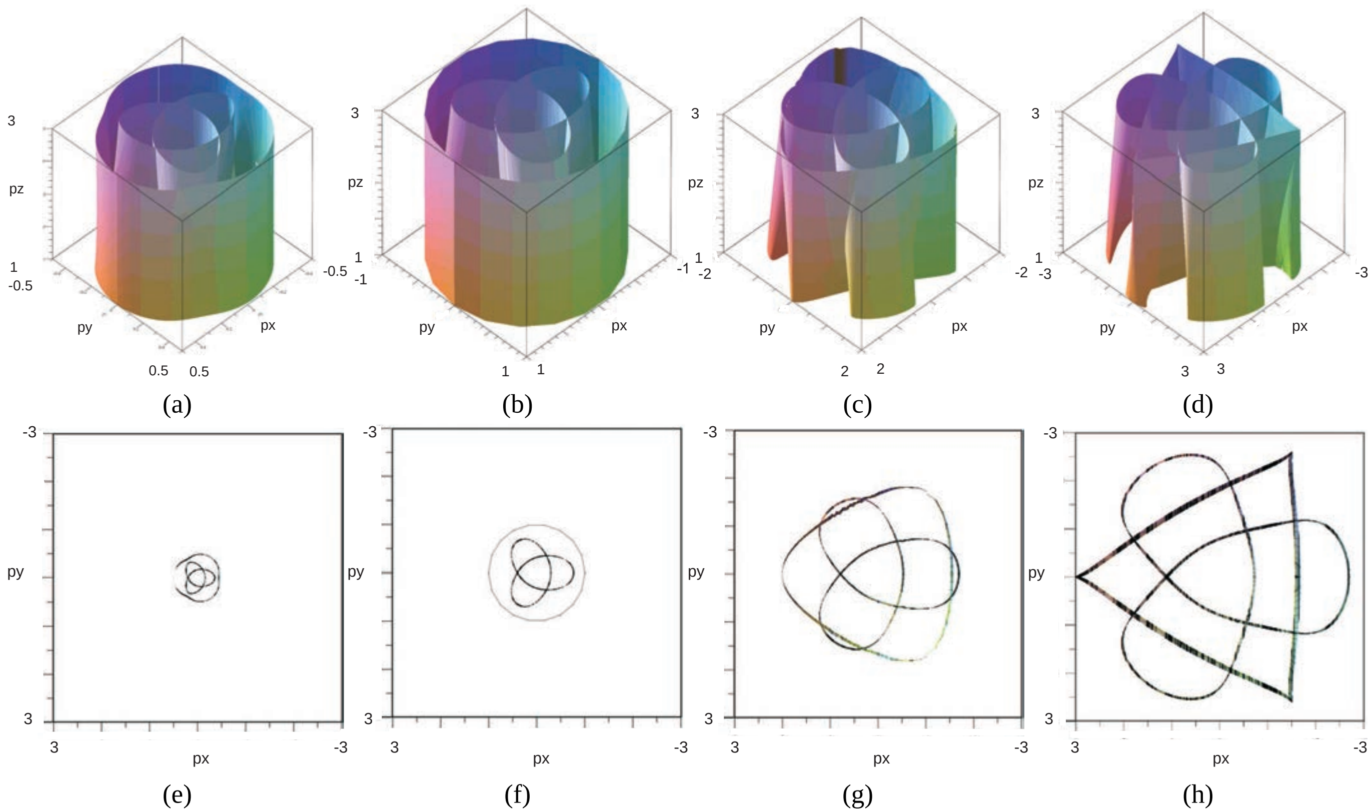


Fig. 4. Projection of parallel singularities in projection space $[p_x, p_y, p_z]$ for OM^2. Variation in the parallel singularities surface for different design parameter k = 0.5 (a), k = 1 (b), k = 2 (c) and k = 3 (d), top view of the parallel singularities surface, k = 0.5 (e), k = 1 (f), k = 2 (g) and k = 3 (h).

workspace analysis of this mechanism is possible, only when the analysis is done in both 2R1T $[p_z, q_2, q_3]$ and 3T $[p_x, p_y, p_z]$ projection spaces for OM_1 and OM_2.

The workspace (resp. Joint space) analysis classifies the number of solutions of the parametric system associated with the Inverse (resp. Direct) Kinematic Problem (IKP). This method was introduced for parallel robots in [12]. The three main steps involved in the analysis are (i) Computation of a subset of the joint space (resp. workspace) where the number of solutions changes: the *Discriminant Variety* [12], (ii) Description of the complementary of the discriminant variety in connected cells: the Generic *Cylindrical Algebraic Decomposition*. and (iii) Connecting the cells belonging to the same connected component in the counterpart of the discriminant variety: *interval comparisons*. Discriminant varieties can be computed using basic and well known tools from computer algebra such as Gröbner bases [21] and a full package computing such objects in a general framework is available in Maple software through the RootFinding[Parametric] package. The CAD implemented in the SIROPA library has been used to compute the aspects into a set of cells where algebraic equations define its boundaries and a sample point in each one [12] for the 2PRR–RPR parallel robot. For example, the CAD can provide a formal decomposition of the joint space in cells where the polynomials (A) and $\det(\mathbf{B})$ have a constant sign and the number for the DKP is constant [22].

The workspace of the robot is a cylinder in the projection space $[\ z,\ \ q_2,\ \ q_3]$ if there are no joint limits on the actuated joints, a is presented in [3]. The workspace analysis can be done by dividing it into a set of aspects. In other words, an aspect is the largest connected region without any singularity of the corresponding operation mode. As there are several solutions for the DKP in the same aspect, non-singular assembly mode trajectories are possible, which is shown in [4, 15] for 3RPS parallel robot. The variation in the workspace boundaries due to the design parameter k for both the operation modes is shown in Fig. 5. Blue and red regions corresponds to the four number of solutions for the IKP for $\det(\mathbf{A}) > 0$ and $\det(\mathbf{A}) < 0$, respectively. These analyses can be useful for the researchers or engineers to select the optimum value for the design parameter such that the parasitic motions can be limited to specific values. From Fig. 5, it can be depicted that as the value of k increases, singularity becomes more complex and there exists a larger area (aspects) without singularities for larger values of k.

5. JOINT SPACE ANALYSIS OF THE 3RPS PARALLEL ROBOT

The joint space analysis allows the characterization of the regions where the number of real solutions for the direct kinematic model is constant. Using CAD, we can do this study on sections of the joint space. The calculation for the full joint space is possible, but the number of cells obtained is too large for the display capabilities of Maple. Without taking into account the notion of operation mode, Figure 7 depicts the regions with 4, 8, 12 or 16 solutions for the DKP. The maximum number for the DKP for each operation mode is 8.

Cuspidal configurations are associated with second-order degeneracies that appear for triply coalesced configurations. These configurations play an important role in the path planning because they are directly linked to the non-singular assembly mode changing trajectories [23–26]. A state of the art for the computation of the cusp points is given in [27]. The variation in the joint space boundaries due to the design parameter k is shown in Figure 6. The slice of joint space is computed for the three different values of ρ_1 and k. For $\rho_1 = 2$, as the value of k increases the cusp points corresponding to the region with 16 numbers of solution for DKP disappears (in Fig. 6(d), 6(e) and 6(f)). The increase in the area of the joint space with an increase in the value of k, shows the more feasible actuation sets compared with the configuration with lesser value of k.

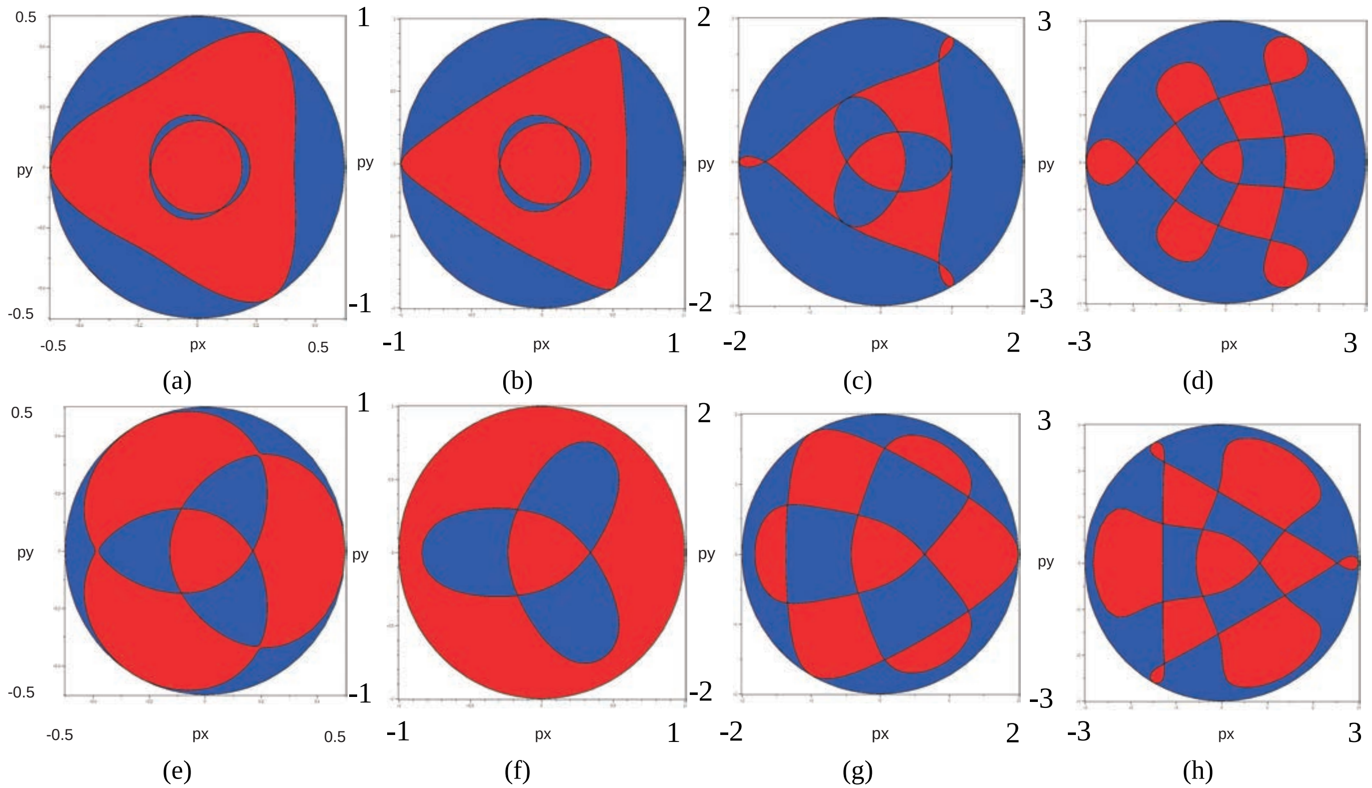


Fig. 5. Slice of workspace for OM^1 with different design parameter and $p_z = 2$, k = 0.5 (a), k = 1 (b), k = 2 (c) and k = 3 (d) for OM^2 k = 0.5 (e), k = 1 (f), k = 2 (g) and k = 3 (h). Blue and red regions corresponds to the four number of solutions for the IKP for $\det(\mathbf{A}) > 0$ and $\det(\mathbf{A}) < 0$, respectively.

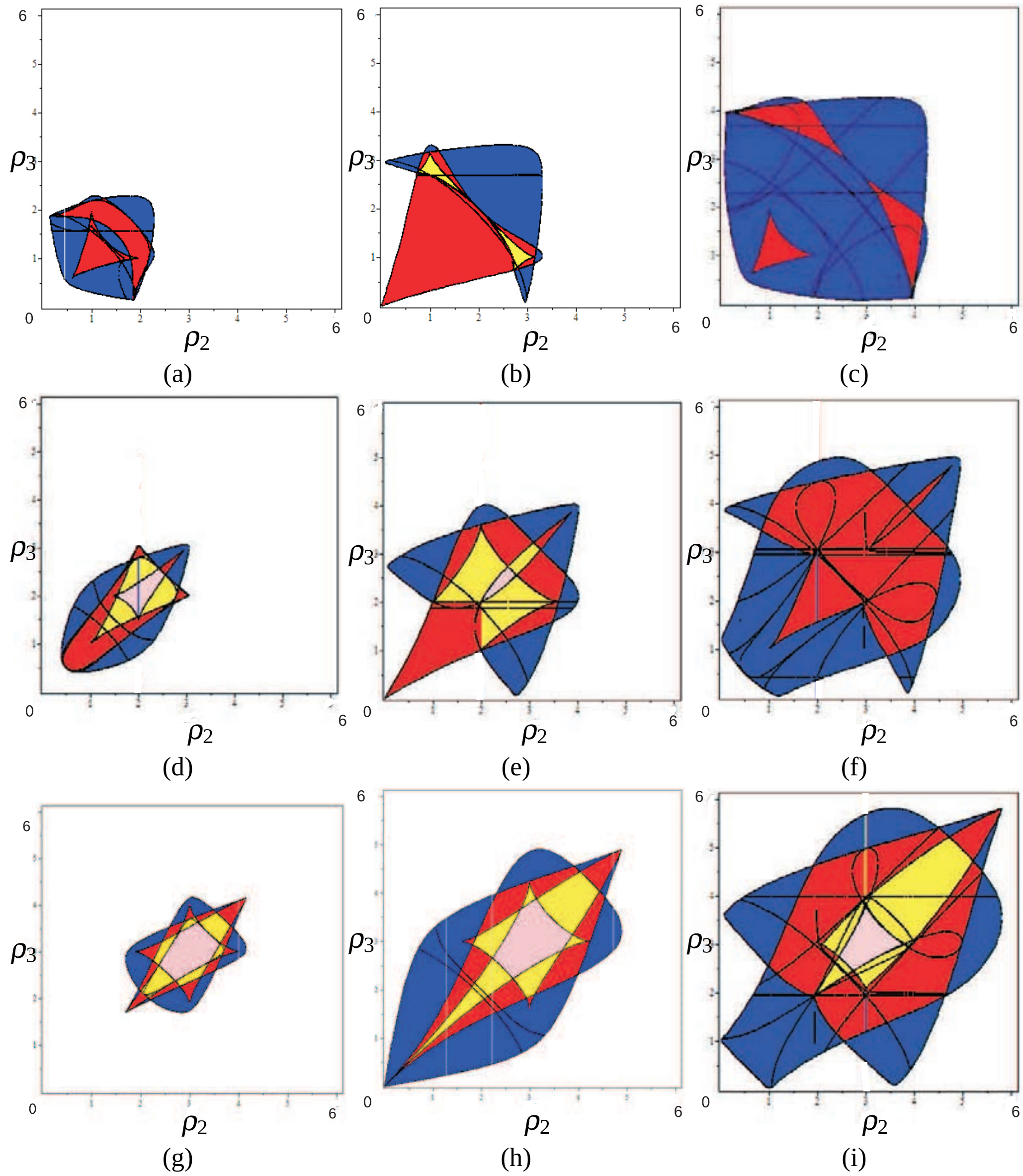


Fig. 6. Slice of the joint space for $\rho_1 = 1$, $k = 0.5$ (a), $\rho_1 = 1$, $k = 1$ (b), $\rho_1 = 1$, $k = 1.5$ (c), $\rho_1 = 2$, $k = 0.5$ (d), $\rho_1 = 2$, $k = 1$ (e), $\rho_1 = 2$, $k = 1.5$ (f), $\rho_1 = 3$, $k = 0.5$ (g), $\rho_1 = 3$, $k = 1$ (h) $\rho_1 = 3$, $k = 1.5$ (i), where the DKP admits four, eight, twelve and sixteen real solutions in the blue, red, yellow and pink region, respectively.

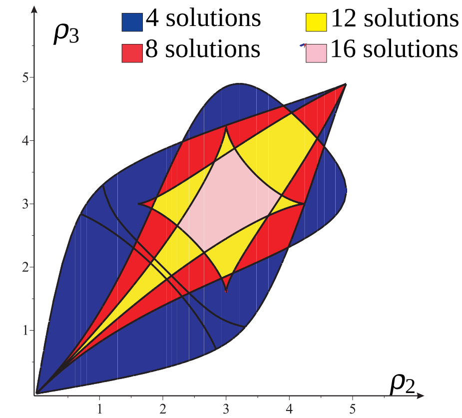


Fig. 7. Slice of the joint space for $\rho_1 = 3$ and the number of solution for DKP for $k = 1$

6. CONCLUSIONS

This work reports the variations in the workspace and the joint space with respect to the design parameter k of the 3-R$\underline{P}$S parallel manipulator. The cylindrical algebraic decomposition method and Gröbner based computations are used to model the workspace and joint space with the parallel singularities in 3T and 2R1T projection spaces, where the orientation of the mobile platform is represented using quaternions. Depending on the design parameter k, three different configurations of the 3-R$\underline{P}$S parallel manipulator are analyzed. A comparative study on the workspace and joint space for different design parameter k, plays an important role in the selection of the manipulator for the specific task or for the trajectory planning.

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