UNIFYING TYPE SYNTHESIS ON FINITE MOTION AND KINEMATIC ANALYSIS ON INSTANTANEOUS MOTION OF PARALLEL MECHANISMS USING SCREW THEORY

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ABSTRACT

It has long been desired to unify the topological model and parametric model, i.e., do type synthesis and kinematics analysis under a unified framework, in the field of mechanisms and robotics. This calls for a mathematical tool that makes analytical description, formulation and operation possible for both finite and instantaneous motions. The authors' previous works show that screw theory is a powerful tool to achieve this goal, which contains finite screws and instantaneous screws to respectively describe finite and instantaneous motions. This paper presents a systematic method to unify type synthesis and kinematic analysis of parallel mechanisms (PMs) under the umbrella of screw theory. Firstly, the topological and parametric models of a PM, its limbs and joints are formulated employing finite and instantaneous screws, respectively. The relationship between these models are built using composition properties and derivative mapping of the two kinds of screws. Then, a general process to do type synthesis and kinematic analysis of PMs is given using these models, in which all the procedures are based upon algebraic and analytical screw operations. Finally, three-DOF spherical PMs (SPMs) are taken as an example to verify the validity of the proposed method and illustrate the whole process.

Keywords: parallel mechanisms; screw theory; type synthesis; kinematic analysis; finite screw.

UNIFIANT SYNTHÈSE DE TYPE MOUVEMENT FINI ET ANALYSE CINÉMATIQUE MOUVEMENT INSTANTANÉ DE MÉCANISMES PARALLÈLES PAR THÉORIE DES VIS

RÉSUMÉ

Les chercheurs désirent depuis longtemps unifier le modèle topologique et le modèle paramétrique, c'est-à-dire faire la synthèse de type et l'analyse cinématique sous un cadre unifié, dans le domaine des mécanismes et de la robotique. Cela nécessite un outil mathématique qui rend possible la description analytique, la formulation et le fonctionnement possibles pour les mouvements finis et instantanés. Les travaux antérieurs des auteurs montrent que la théorie des vis est un outil puissant pour atteindre cet objectif, qui contient des vis finies et des vis instantanées pour décrire respectivement des mouvements finis et instantanés. Cet article présente une méthode systématique pour unifier la synthèse de type et l'analyse cinématique des mécanismes parallèles (PMs) sous l'égide de la théorie des vis. Premièrement, les modèles topologiques et paramétriques d'un PM, de ses membres et de ses articulations sont formulés en utilisant des vis finies et instantanées, respectivement. La relation entre ces modèles est construite à partir des propriétés de composition et de la cartographie dérivée des deux types de vis. Ensuite, un procédé général de synthèse de type et d'analyse cinématique des PM est donné en utilisant ces modèles, dans lesquels toutes les procédures sont basées sur des opérations de vis algébriques et analytiques. Enfin, on prend comme exemple un PM sphérique à trois DOF (SPM) pour vérifier la validité de la méthode proposée et illustrer l'ensemble du processus.

Mots-clés : mécanismes parallèles; théorie des vis; synthèse de types; analyse cinématique; vis fini.

1 INTRODUCTION

It has long been desired to unify type synthesis and kinematic analysis of robotic mechanisms into a general process using a consistent theoretical package. In this fundamental and challenging issue, a prerequisite and essential step is to select or develop an effective mathematical tool, which enables the analytical description, formulation and operation of both finite and instantaneous motions to be implemented, resulting in topological and parametric models that are connected algebraically. At present, there are three available mathematical tools at hand, i.e. matrix group, dual quaternion and screw theory.

The matrix group was proposed by Lie and introduced to describe the rigid body motion by Klein in Erlangen program. Using Lie subgroups of SE(3) and its composite manifolds to describe the finite motions of joint, serial and parallel mechanisms [1], Hervé [2] gave a method to formulate topological models and did type synthesis of parallel mechanisms (PMs). Li and Hervé [3, 4] synthesized many PMs with different motion patterns employing matrix groups and manifolds. By utilizing Lie algebra se(3) of SE(3) to describe instantaneous motions, Brockett [5] applied the exponential mapping between SE(3) and se(3) to relating topological model for type synthesis and parametric model for kinematic analysis of serial mechanisms. This work was extended and applied to PMs and other kinds of mechanisms. It should be noted that two barriers are encountered when using matrix groups for finite motion composition to formulate topological models. One barrier comes from the fact that matrix group representations of finite motions cannot be directly expressed by Chasles' axis with angular and/or linear displacement about the axis, leading to a complicated description of rigid body motion. The other barrier arises from the inability to algebraically implement a finite motion composition for matrix form by using Baker-Campbell-Hausdorff formula. Thus, even though parametric models can be no doubt given by Lie algebra se(3), topological models of many PMs cannot precisely be obtained using the existing matrix group based method because they can no longer be formulated by the group products of a few Lie subgroups of SE(3).

Dual quaternion representation of rigid body motion can be traced back to description of rotations utilizing Euler's four-square identity, Euler-Rodrigues parameters and Hamilton quaternions. As far as the authors know, Perez and McCarthy [6] are possibly the first to use dual quaternions to do finite and instantaneous motion analyses of serial kinematic chains, resulting in kinematic synthesis and simulation models. They used unit dual quaternions and unit pure dual quaternions to respectively describe finite and instantaneous motions, because the algebraic structure of the former is a double cover of SE(3) whose Lie algebra in turn constitutes the latter. By means of group theory, Selig [7] and Dai [8] investigated algebraic properties of the exponential and Cayley mappings between unit dual quaternions and unit pure dual quaternions, leading to a clear indication of the relationship between finite and instantaneous models. Then, the dual quaternions representation was extended to deal with dynamics problems through applying high-dimensional Clifford algebra. Although unit dual quaternions can describe both finite and instantaneous motions and their relationship, they are not the simplest forms of rigid body motions. The redundancy in dual quaternion representation may cause complexity in analytical operations of finite motions. Furthermore, the Rodrigues formula with dual angles is not the simplest form of the Baker-Campbell-Hausdorff formula in composition of finite motions.

Screw theory was firstly proposed by Ball and has been developed to be a powerful and effective tool in fields of mechanisms and robotics. As shown in the authors' previous work [9-11], finite and instantaneous screws can describe finite and instantaneous motions in concise and non-redundant forms and can be analytically composited. The algebraic structures of the sets of these two kinds of screws were revealed and the derivative mapping between them was built. All these achievements show that screw theory has the potential to unify type synthesis and kinematic analysis into a general and consistent process, which can overcome the shortcomings of the above matrix group and dual quaternion based methods.

Mainly drawing on screw theory, this paper proposes a systematic method to unify type synthesis in finite motion level and kinematic analysis in instantaneous level of PMs. The paper is organized as follows. Having a brief review of the state-of-the-art of the existing methods based upon different mathematical tools to uniformly describe finite and instantaneous motions in Section 1, Section 2 presents the new method to formulate topological and parametric models of a PM, its limbs and joints using finite and instantaneous screws. In Section 3, a general process based upon algebraic and analytical screw operations to do type synthesis and kinematic analysis of PMs is given using these models. Three-DOF spherical PM (SPM) is taken as an example to verify the validity of the proposed method and illustrate the whole process in Section 4 before the conclusions are drawn in Section 5.

2 SCREW THEORY BASED TOPOLOGICAL AND PARAMETRIC MODELS

2.1 Finite and Instantaneous Motions of a Rigid Body

According to Chasles theorem, a finite motion of a rigid body from initial pose (pose 1) to arbitrary pose (pose 2) can be equivalently regarded as a rotation about the Chasles axis followed by a translation along that axis as shown in Fig. 1(a), which can be described by a finite screw $\boldsymbol{S}_f$ in quasi-vector [9] form as

$$\boldsymbol{S}_f = 2\tan\frac{\theta}{2}\begin{pmatrix} \boldsymbol{s}_f \\ \boldsymbol{r}_f \times \boldsymbol{s}_f \end{pmatrix} + t\begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{s}_f \end{pmatrix} \tag{1}$$

where $\boldsymbol{s}_f$ and $\boldsymbol{r}_f$ denote the unit vector and position vector of the finite motion axis, θ and t are the angular displacement about and linear displacement along that axis with respect to the initial pose.

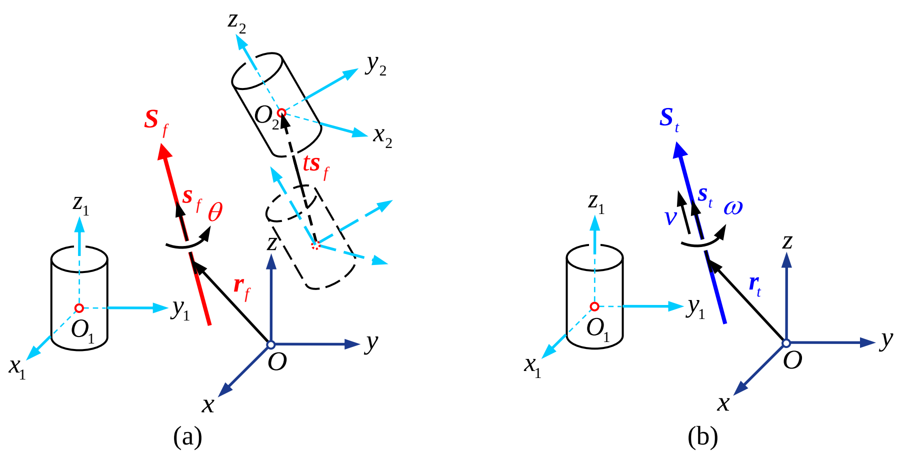

Fig. 1. Finite and instantaneous motion of a rigid body: (a) finite motion; (b) instantaneous motion.

The derivative of $\boldsymbol{S}_f$ at the initial pose ($\theta = 0$ and $t = 0$) where the finite motion axis is coincident with the instantaneous motion axis at the instant has been derived as [9]

$$\dot{\boldsymbol{S}}_f\Big|_{\substack{\theta=0\\t=0}} = \dot{\theta}\begin{pmatrix} \boldsymbol{s}_t \\ \boldsymbol{r}_t \times \boldsymbol{s}_t \end{pmatrix} + \dot{t}\begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{s}_t \end{pmatrix} \tag{2}$$

which is exactly an instantaneous screw in vector form as shown in Fig. 1(b)

$$\boldsymbol{S}_t = \dot{\boldsymbol{S}}_f\Big|_{\substack{\theta=0\\t=0}} = \omega\begin{pmatrix} \boldsymbol{s}_t \\ \boldsymbol{r}_t \times \boldsymbol{s}_t \end{pmatrix} + v\begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{s}_t \end{pmatrix} = \begin{pmatrix} \boldsymbol{\omega} \\ \boldsymbol{v} \end{pmatrix} \tag{3}$$

where ω and v are the angular velocity and linear velocity about/along the instantaneous motion axis. Eq. (3) indicates that there exists a derivative mapping between finite and instantaneous screws. Using these two kinds of screws and their relationship, the topological and parametric models of one-DOF joints, a limb and a PM will be formulated.

2.2 Topological and Parametric Models of One-DOF Joints

In Eq. (1) and Eq. (3), θ and t, ω and v can be regarded as variables. Thus, the finite and instantaneous motions generated by one-DOF joints, a limb and a PM can be described by finite screw sets and instantaneous screw systems. In this way, the topological and parametric models can be consistently formulated using screw theory.

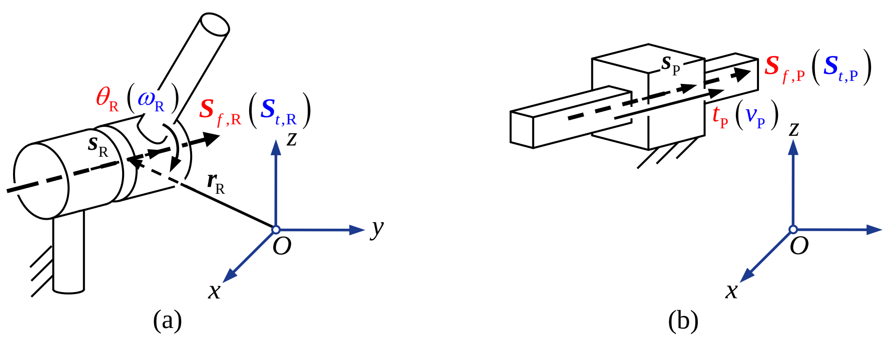

Fig. 2. Finite and instantaneous motions of one-DOF joints: (a) R joint; (b) P joint.

As shown in Fig. 2(a-b), topological and parametric models of a revolute (R) joint and a prismatic (P) joint can be obtained as

(a) R joint:

$$\left\{\boldsymbol{S}_{f,\mathrm{R}}\right\} = \left\{2\tan\frac{\theta_{\mathrm{R}}}{2}\begin{pmatrix}\boldsymbol{s}_{\mathrm{R}} \\ \boldsymbol{r}_{\mathrm{R}}\times\boldsymbol{s}_{\mathrm{R}}\end{pmatrix}\right\} \tag{4}$$

$$\left\{\boldsymbol{S}_{t,\mathrm{R}}\right\} = \mathrm{span}\left\{\omega_{\mathrm{R}}\begin{pmatrix}\boldsymbol{s}_{\mathrm{R}} \\ \boldsymbol{r}_{\mathrm{R}}\times\boldsymbol{s}_{\mathrm{R}}\end{pmatrix}\right\} \tag{5}$$

(a) P joint:

$$\left\{\boldsymbol{S}_{f,\mathrm{P}}\right\} = \left\{t_{\mathrm{P}}\begin{pmatrix}\mathbf{0} \\ \boldsymbol{s}_{\mathrm{P}}\end{pmatrix}\right\} \tag{6}$$

$$\left\{\boldsymbol{S}_{t,\mathrm{P}}\right\} = \mathrm{span}\left\{v_{\mathrm{P}}\begin{pmatrix}\mathbf{0} \\ \boldsymbol{s}_{\mathrm{P}}\end{pmatrix}\right\} \tag{7}$$

where $\left\{\boldsymbol{S}_{f,\mathrm{R}}\right\}$ ($\left\{\boldsymbol{S}_{f,\mathrm{P}}\right\}$) and $\left\{\boldsymbol{S}_{t,\mathrm{R}}\right\}$ ($\left\{\boldsymbol{S}_{t,\mathrm{P}}\right\}$) are finite screw sets and instantaneous screw systems generated by the R (P) joint, the denotations of $\boldsymbol{s}_{\mathrm{R}}$, $\boldsymbol{r}_{\mathrm{R}}$, θ_{R}, ω_{R} and $\boldsymbol{s}_{\mathrm{P}}$, t_{P}, v_{P} can be referred to the symbols in Eq. (1) and Eq. (3). According to the derivations in [9] and Eq. (3), the derivative mapping between the topological and parametric models of one-DOF R and P joint can be built as

$$\left\{\dot{\boldsymbol{S}}_{f,\mathrm{R}}\Big|_{\theta_{\mathrm{R}}=0}\right\} = \left\{\boldsymbol{S}_{t,\mathrm{R}}\right\} \tag{8}$$

$$\left\{ \dot{\boldsymbol{S}}_{f,\mathrm{P}} \Big|_{t_{\mathrm{P}}=0} \right\} = \left\{ \boldsymbol{S}_{t,\mathrm{P}} \right\} \tag{9}$$

2.3 Topological and Parametric Models of a Limb

Considering a limb (serial kinematic chain) constituted by *n* one-DOF joints as shown in Fig. 3, the finite and instantaneous motions realized by its end-effector are the compositions of those generated by all its joints. As finite screws can be analytically composited by screw triangle product "$\triangle$" [9] and instantaneous screws can be linearly added, the topological and parametric models of the limb can be formulated as follows.

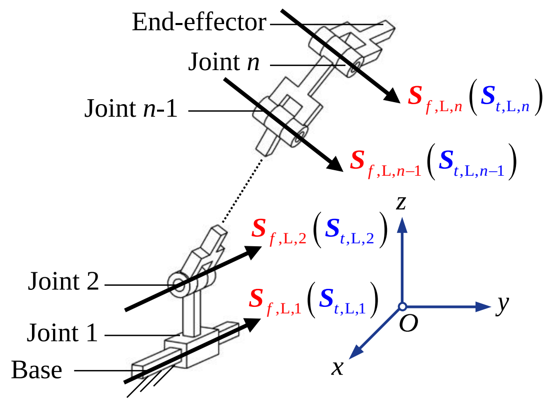

Fig. 3. Finite and instantaneous motions of a limb.

$$\left\{ \boldsymbol{S}_{f,\mathrm{L}} \right\} = \left\{ \boldsymbol{S}_{f,\mathrm{L},n} \triangle \boldsymbol{S}_{f,\mathrm{L},n-1} \triangle \cdots \triangle \boldsymbol{S}_{f,\mathrm{L},1} \right\} \tag{10}$$

$$\left\{ \boldsymbol{S}_{t,\mathrm{L}} \right\} = \mathrm{span} \left\{ \boldsymbol{S}_{t,\mathrm{L},n}, \boldsymbol{S}_{t,\mathrm{L},n-1}, \cdots, \boldsymbol{S}_{t,\mathrm{L},1} \right\} \tag{11}$$

where $\left\{ \boldsymbol{S}_{f,\mathrm{L}} \right\}$ and $\left\{ \boldsymbol{S}_{t,\mathrm{L}} \right\}$ are the finite screw sets and instantaneous screw systems realized by the end-effector of the limb, $\left\{ \boldsymbol{S}_{f,\mathrm{L},k} \right\}$ and $\left\{ \boldsymbol{S}_{t,\mathrm{L},k} \right\}$ ($k = 1, 2, \cdots, n$) are the finite screw sets and instantaneous screw systems generated by the *k*th joint in the limb. The expressions for $\left\{ \boldsymbol{S}_{f,\mathrm{L},k} \right\}$ and $\left\{ \boldsymbol{S}_{t,\mathrm{L},k} \right\}$ can be referred to Eqs. (6-9).

According to the associativity and derivative laws of screw triangle products [9], the following equation can be obtained

$$\dot{\boldsymbol{S}}_{f,\mathrm{L}} \Big|_{\substack{\theta_k = 0 \\ t_k = 0}, \, k=1,2,\cdots,n} = \sum_{k=1}^{n} \dot{\boldsymbol{S}}_{f,\mathrm{L},k} \Big|_{\substack{\theta_k = 0 \\ t_k = 0}, \, k=1,2,\cdots,n} = \sum_{k=1}^{n} \boldsymbol{S}_{t,\mathrm{L},k} \tag{12}$$

From above relationship, the derivative mapping between the topological and parametric models of the limb can be built as

$$\left\{ \dot{\boldsymbol{S}}_{f,\mathrm{L}} \Big|_{\substack{\theta_k = 0 \\ t_k = 0}, \, k=1,2,\cdots,n} \right\} = \left\{ \boldsymbol{S}_{t,\mathrm{L}} \right\} \tag{13}$$

2.4 Topological and Parametric Models of a PM

As shown in Fig. 4, a PM is composed of l limbs. Because all the l limbs share the same end-effector, i.e., the moving platform of the PM, the finite and instantaneous motions realized by the moving platform are the intersection of those generated by all these limbs. In this way, we can formulate the topological and parametric models of the PM as

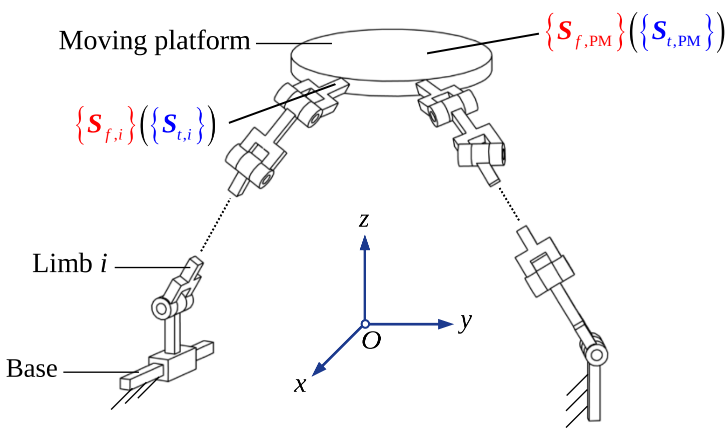

Fig. 4. Finite and instantaneous motions of a PM.

$$\{\boldsymbol{S}_{f,\mathrm{PM}}\} = \{\boldsymbol{S}_{f,1}\} \cap \{\boldsymbol{S}_{f,2}\} \cap \cdots \cap \{\boldsymbol{S}_{f,l}\} \tag{14}$$

$$\{\boldsymbol{S}_{t,\mathrm{PM}}\} = \{\boldsymbol{S}_{t,1}\} \cap \{\boldsymbol{S}_{t,2}\} \cap \cdots \cap \{\boldsymbol{S}_{t,l}\} \tag{15}$$

where $\{\boldsymbol{S}_{f,i}\}$ and $\{\boldsymbol{S}_{t,i}\}$ ($i = 1, 2, \cdots, l$) are the finite screw sets and instantaneous screw systems generated by the ith limb. Their expressions can be formulatd using Eq. (10) and Eq. (11). Through regarding the expressions of $\{\boldsymbol{S}_{f,i}\}$ as simultaneous equations, $\{\boldsymbol{S}_{f,\mathrm{PM}}\}$ can be obtained by analytical derivations. Because $\{\boldsymbol{S}_{t,i}\}$ are linear spaces, $\{\boldsymbol{S}_{t,\mathrm{PM}}\}$ can be solved using linear algebra.

The differential of Eq. (14) at the initial pose of the PM is

$$\left\{ \dot{\boldsymbol{S}}_{f,\mathrm{PM}} \Big|_{\substack{\theta_{i,k}=0 \\ t_{i,k}=0}, \substack{i=1,2,\cdots,l \\ k=1,2,\cdots,n_i}} \right\} = \left\{ \dot{\boldsymbol{S}}_{f,1} \Big|_{\substack{\theta_{1,k}=0 \\ t_{1,k}=0}, k=1,2,\cdots,n_1} \right\} \cap \left\{ \dot{\boldsymbol{S}}_{f,2} \Big|_{\substack{\theta_{2,k}=0 \\ t_{2,k}=0}, k=1,2,\cdots,n_2} \right\} \cap \cdots \cap \left\{ \dot{\boldsymbol{S}}_{f,l} \Big|_{\substack{\theta_{l,k}=0 \\ t_{l,k}=0}, k=1,2,\cdots,n_l} \right\} \tag{16}$$

Substituting Eq. (13) to Eq. (16), yields

$$\left\{ \dot{\boldsymbol{S}}_{f,\mathrm{PM}} \Big|_{\substack{\theta_{i,k}=0 \\ t_{i,k}=0}, \substack{i=1,2,\cdots,l \\ k=1,2,\cdots,n_i}} \right\} = \{\boldsymbol{S}_{t,1}\} \cap \{\boldsymbol{S}_{t,2}\} \cap \cdots \cap \{\boldsymbol{S}_{t,l}\} \tag{17}$$

Hence, the derivative mapping between the topological and parametric models of the PM can be built using Eq. (15) and Eq. (17).

$$\left\{ \dot{\boldsymbol{S}}_{f,\mathrm{PM}} \Big|_{\substack{\theta_{i,k}=0 \\ t_{i,k}=0}, \substack{i=1,2,\cdots,l \\ k=1,2,\cdots,n_i}} \right\} = \{\boldsymbol{S}_{t,\mathrm{PM}}\} \tag{18}$$

Using finite and instantaneous screws to describe finite and instantaneous motions of a rigid body, one-DOF joints, a limb and a PM, the topological and parametric models of a PM, its limbs and joints are formulated. In this manner, type synthesis and kinematic analysis of PMs can be carried out under the concise and consistent theoretical package of screw theory. Unlike methods purely using instantaneous screws [12, 13], the method proposed in this paper do type synthesis of PMs in finite motion level.

3 TYPE SYNTHESIS AND KINEMATIC ANALYSIS BY SCREW THEORY

Having the topological and parametric models formulated by screw theory at hand, type synthesis and kinematic analysis can be unified in the following process.
Step 1: Describe the expected motion pattern of PMs using finite screw set.
Step 2: Formulate the whole limb bonds based upon Eq. (14) and obtain all the feasible limb structures with proper arrangements of joints using Eq. (10).
Step 3: Derive the assembly conditions and actuation arrangements using Eq. (14), synthesize all the PMs with expected motion pattern.
Step 4: Given a desired PM with specific topological structure, obtain the instantaneous screw system of it based upon Eq. (17) and Eq. (18).
Step 5: Formulate Jacobian matrix of the PM which is ready for applying in velocity, acceleration, stiffness and dynamic analysis.
As shown in Fig. 5, all the procedures in this process are completely based upon algebraic and analytical operations which have been given in Section 2.

In the next section, an example will be given to illustrate this process more clearly.

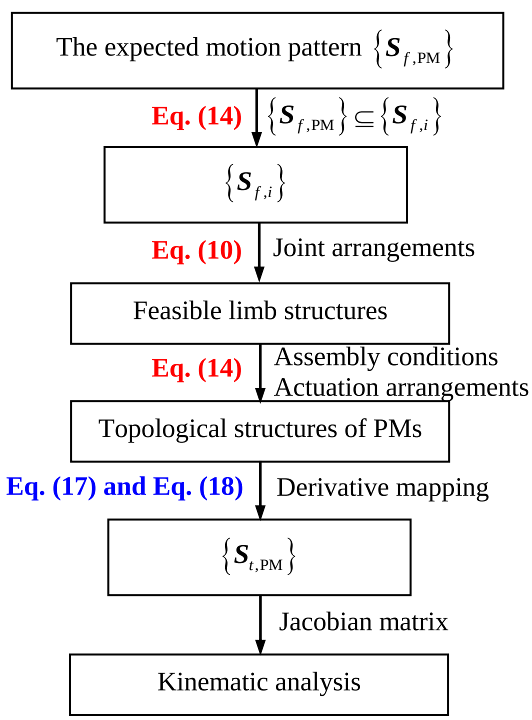

Fig. 5. Type synthesis and kinematic analysis of PMs using screw theory.

4 EXAMPLE

In this section, type synthesis and kinematic analysis of SPMs will be given to shown the validity of the screw theory based method proposed in Sections 2 and 3.

4.1 Type synthesis of SPMs

The expected motion pattern of a SPM can be formulated by finite screw set as

$$\left\{ \boldsymbol{S}_{f,\mathrm{SPM}} \right\} = \left\{ \left. \begin{pmatrix} \boldsymbol{s} \\ \boldsymbol{r}_O \times \boldsymbol{s} \end{pmatrix} \right| \boldsymbol{s} \in \mathbb{R}^3 \right\} \tag{19}$$

where point O is the spherical center of the SPM. Using the properties of screw triangle product [9], Eq. (19) can be rewritten as

$$\left\{ \boldsymbol{S}_{f,\mathrm{SPM}} \right\} = \left\{ 2\tan\frac{\theta_c}{2}\begin{pmatrix} \boldsymbol{s}_c \\ \boldsymbol{r}_O \times \boldsymbol{s}_c \end{pmatrix} \triangle 2\tan\frac{\theta_b}{2}\begin{pmatrix} \boldsymbol{s}_b \\ \boldsymbol{r}_O \times \boldsymbol{s}_b \end{pmatrix} \triangle 2\tan\frac{\theta_a}{2}\begin{pmatrix} \boldsymbol{s}_a \\ \boldsymbol{r}_O \times \boldsymbol{s}_a \end{pmatrix} \right\} \tag{20}$$

where $\boldsymbol{s}_a$, $\boldsymbol{s}_b$ and $\boldsymbol{s}_c$ are three independent unit vectors. Using Eq. (10), it is easy to see that Eq. (20) can be equivalently generated by a serial kinematic chain $\mathrm{R}_a\mathrm{R}_b\mathrm{R}_c$ constituted by three R joints, whose axes pass through the fixed point O and are not in the same plane.

According to Eq. (14), a limb bond $\left\{ \boldsymbol{S}_{f,i} \right\}$ of SPM should be a subset of Eq. (20). Thus, six standard $\left\{ \boldsymbol{S}_{f,i} \right\}$ can be formulated through adding zero, one or two translational/rotational factors.

$$\left\{ \boldsymbol{S}_{f,l_{\mathrm{I}}} \right\} = \left\{ 2\tan\frac{\theta_c}{2}\begin{pmatrix} \boldsymbol{s}_c \\ \boldsymbol{r}_O \times \boldsymbol{s}_c \end{pmatrix} \triangle 2\tan\frac{\theta_b}{2}\begin{pmatrix} \boldsymbol{s}_b \\ \boldsymbol{r}_O \times \boldsymbol{s}_b \end{pmatrix} \triangle 2\tan\frac{\theta_a}{2}\begin{pmatrix} \boldsymbol{s}_a \\ \boldsymbol{r}_O \times \boldsymbol{s}_a \end{pmatrix} \right\} \tag{21}$$

$$\left\{ \boldsymbol{S}_{f,l_{\mathrm{II}}} \right\}_{\mathrm{P}} = \left\{ 2\tan\frac{\theta_c}{2}\begin{pmatrix} \boldsymbol{s}_c \\ \boldsymbol{r}_O \times \boldsymbol{s}_c \end{pmatrix} \triangle 2\tan\frac{\theta_b}{2}\begin{pmatrix} \boldsymbol{s}_b \\ \boldsymbol{r}_O \times \boldsymbol{s}_b \end{pmatrix} \triangle 2\tan\frac{\theta_a}{2}\begin{pmatrix} \boldsymbol{s}_a \\ \boldsymbol{r}_O \times \boldsymbol{s}_a \end{pmatrix} \triangle t_1 \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{s}_1 \end{pmatrix} \right\} \tag{22}$$

$$\left\{ \boldsymbol{S}_{f,l_{\mathrm{III}}} \right\}_{\mathrm{R}} = \left\{ 2\tan\frac{\theta_c}{2}\begin{pmatrix} \boldsymbol{s}_c \\ \boldsymbol{r}_O \times \boldsymbol{s}_c \end{pmatrix} \triangle 2\tan\frac{\theta_b}{2}\begin{pmatrix} \boldsymbol{s}_b \\ \boldsymbol{r}_O \times \boldsymbol{s}_b \end{pmatrix} \triangle 2\tan\frac{\theta_a}{2}\begin{pmatrix} \boldsymbol{s}_a \\ \boldsymbol{r}_O \times \boldsymbol{s}_a \end{pmatrix} \triangle 2\tan\frac{\theta_1}{2}\begin{pmatrix} \boldsymbol{s}_1 \\ \boldsymbol{r}_1 \times \boldsymbol{s}_1 \end{pmatrix} \right\} \tag{23}$$

$$\left\{ \boldsymbol{S}_{f,l_{\mathrm{IV}}} \right\}_{\mathrm{PP}} = \left\{ 2\tan\frac{\theta_c}{2}\begin{pmatrix} \boldsymbol{s}_c \\ \boldsymbol{r}_O \times \boldsymbol{s}_c \end{pmatrix} \triangle 2\tan\frac{\theta_b}{2}\begin{pmatrix} \boldsymbol{s}_b \\ \boldsymbol{r}_O \times \boldsymbol{s}_b \end{pmatrix} \triangle 2\tan\frac{\theta_a}{2}\begin{pmatrix} \boldsymbol{s}_a \\ \boldsymbol{r}_O \times \boldsymbol{s}_a \end{pmatrix} \triangle t_2 \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{s}_2 \end{pmatrix} \triangle t_1 \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{s}_1 \end{pmatrix} \right\} \tag{24}$$

$$\left\{ \boldsymbol{S}_{f,l_{\mathrm{V}}} \right\}_{\mathrm{RR}} = \left\{ \begin{array}{l} 2\tan\frac{\theta_c}{2}\begin{pmatrix} \boldsymbol{s}_c \\ \boldsymbol{r}_O \times \boldsymbol{s}_c \end{pmatrix} \triangle 2\tan\frac{\theta_b}{2}\begin{pmatrix} \boldsymbol{s}_b \\ \boldsymbol{r}_O \times \boldsymbol{s}_b \end{pmatrix} \triangle 2\tan\frac{\theta_a}{2}\begin{pmatrix} \boldsymbol{s}_a \\ \boldsymbol{r}_O \times \boldsymbol{s}_a \end{pmatrix} \\ \triangle 2\tan\frac{\theta_2}{2}\begin{pmatrix} \boldsymbol{s}_2 \\ \boldsymbol{r}_2 \times \boldsymbol{s}_2 \end{pmatrix} \triangle 2\tan\frac{\theta_1}{2}\begin{pmatrix} \boldsymbol{s}_1 \\ \boldsymbol{r}_1 \times \boldsymbol{s}_1 \end{pmatrix} \end{array} \right\} \tag{25}$$

$$\left\{ \boldsymbol{S}_{f,l_{\mathrm{VI}}} \right\}_{\mathrm{PR}} = \left\{ 2\tan\frac{\theta_c}{2}\begin{pmatrix} \boldsymbol{s}_c \\ \boldsymbol{r}_O \times \boldsymbol{s}_c \end{pmatrix} \triangle 2\tan\frac{\theta_b}{2}\begin{pmatrix} \boldsymbol{s}_b \\ \boldsymbol{r}_O \times \boldsymbol{s}_b \end{pmatrix} \triangle 2\tan\frac{\theta_a}{2}\begin{pmatrix} \boldsymbol{s}_a \\ \boldsymbol{r}_O \times \boldsymbol{s}_a \end{pmatrix} \triangle 2\tan\frac{\theta_2}{2}\begin{pmatrix} \boldsymbol{s}_2 \\ \boldsymbol{r}_2 \times \boldsymbol{s}_2 \end{pmatrix} \triangle t_1 \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{s}_1 \end{pmatrix} \right\} \tag{26}$$

which correspond to six standard limb structures obtained by adding zero, one or two P/R joints to the limb $\mathrm{R}_a\mathrm{R}_b\mathrm{R}_c$, i.e., $\mathrm{R}_a\mathrm{R}_b\mathrm{R}_c$, $\mathrm{P}_1\mathrm{R}_a\mathrm{R}_b\mathrm{R}_c$, $\underline{\underline{\mathrm{R}}}_1\mathrm{R}_a\mathrm{R}_b\mathrm{R}_c$, $\mathrm{P}_1\mathrm{P}_2\mathrm{R}_a\mathrm{R}_b\mathrm{R}_c$, $\underline{\underline{\mathrm{R}}}_1\underline{\underline{\mathrm{R}}}_2\mathrm{R}_a\mathrm{R}_b\mathrm{R}_c$, $\mathrm{P}_1\underline{\underline{\mathrm{R}}}_2\mathrm{R}_a\mathrm{R}_b\mathrm{R}_c$. It should be noted that $\underline{\underline{\mathrm{R}}}$ denotes a R joint whose axis do not pass through point O.

By utilizing the properties of screw triangle product, it can be proved that arbitrarily changing the joint locations in these six standard limbs will always results in feasible limb structures whose limb bond $\left\{ \boldsymbol{S}_{f,i} \right\}$ satisfying the condition $\left\{ \boldsymbol{S}_{f,\mathrm{SPM}} \right\} \subseteq \left\{ \boldsymbol{S}_{f,i} \right\}$.

For example, a four-DOF limb structure $\mathrm{R}_a\underline{\underline{\mathrm{R}}}_1\mathrm{R}_b\mathrm{R}_c$ can be obtained by changing the location of $\underline{\underline{\mathrm{R}}}_1$ in $\underline{\underline{\mathrm{R}}}_1\mathrm{R}_a\mathrm{R}_b\mathrm{R}_c$. According to Eq. (10), $\left\{\boldsymbol{S}_{f,i}\right\}$ generated by $\mathrm{R}_a\underline{\underline{\mathrm{R}}}_1\mathrm{R}_b\mathrm{R}_c$ can be formulated as

$$\left\{\boldsymbol{S}'_{f,l_{\mathrm{III}}}\right\}_{\mathrm{R}} = \left\{2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_O\times\boldsymbol{s}_c\end{pmatrix}\triangle 2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\ \boldsymbol{r}_O\times\boldsymbol{s}_b\end{pmatrix}\triangle 2\tan\frac{\theta_1}{2}\begin{pmatrix}\boldsymbol{s}_1\\ \boldsymbol{r}_1\times\boldsymbol{s}_1\end{pmatrix}\triangle 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_O\times\boldsymbol{s}_a\end{pmatrix}\right\} \tag{27}$$

Eq. (27) can be rewritten as

$$\left\{\boldsymbol{S}'_{f,l_{\mathrm{III}}}\right\}_{\mathrm{R}} = \left\{\begin{array}{l} 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_O\times\boldsymbol{s}_c\end{pmatrix}\triangle 2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\ \boldsymbol{r}_O\times\boldsymbol{s}_b\end{pmatrix}\triangle 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_O\times\boldsymbol{s}_a\end{pmatrix} \\ \triangle 2\tan\frac{\theta_1}{2}\begin{pmatrix}\exp\left(\theta_a\tilde{\boldsymbol{s}}_a\right)\boldsymbol{s}_1\\ \left(\boldsymbol{r}_O+\exp\left(\theta_a\tilde{\boldsymbol{s}}_a\right)\left(\boldsymbol{r}_1-\boldsymbol{r}_O\right)\right)\times\left(\exp\left(\theta_a\tilde{\boldsymbol{s}}_a\right)\boldsymbol{s}_1\right)\end{pmatrix}\end{array}\right\} \tag{28}$$

The first three factors in Eq. (28) are the same as those in $\left\{\boldsymbol{S}_{f,\mathrm{SPM}}\right\}$. Hence, $\left\{\boldsymbol{S}_{f,\mathrm{SPM}}\right\}\subseteq\left\{\boldsymbol{S}'_{f,l_{\mathrm{III}}}\right\}_{\mathrm{R}}$ is satisfied, which indicates that $\mathrm{R}_a\underline{\underline{\mathrm{R}}}_1\mathrm{R}_b\mathrm{R}_c$ is a feasible limb structure of SPMs.

Another example is a five-DOF limb structure $\mathrm{R}_a\mathrm{P}_1\underline{\underline{\mathrm{R}}}_2\mathrm{R}_b\mathrm{R}_c$ obtained from $\mathrm{P}_1\underline{\underline{\mathrm{R}}}_2\mathrm{R}_a\mathrm{R}_b\mathrm{R}_c$. Its $\left\{\boldsymbol{S}_{f,i}\right\}$ is

$$\left\{\boldsymbol{S}'_{f,l_{\mathrm{VI}}}\right\}_{\mathrm{PR}} = \left\{2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_O\times\boldsymbol{s}_c\end{pmatrix}\triangle 2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\ \boldsymbol{r}_O\times\boldsymbol{s}_b\end{pmatrix}\triangle 2\tan\frac{\theta_2}{2}\begin{pmatrix}\boldsymbol{s}_2\\ \boldsymbol{r}_2\times\boldsymbol{s}_2\end{pmatrix}\triangle t_1\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_1\end{pmatrix}\triangle 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_O\times\boldsymbol{s}_a\end{pmatrix}\right\} \tag{29}$$

Eq. (29) can be rewritten as

$$\left\{\boldsymbol{S}'_{f,l_{\mathrm{VI}}}\right\}_{\mathrm{PR}} = \left\{\begin{array}{l} 2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_O\times\boldsymbol{s}_c\end{pmatrix}\triangle 2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\ \boldsymbol{r}_O\times\boldsymbol{s}_b\end{pmatrix}\triangle 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_O\times\boldsymbol{s}_a\end{pmatrix} \\ \triangle 2\tan\frac{\theta_2}{2}\begin{pmatrix}\exp\left(\theta_a\tilde{\boldsymbol{s}}_a\right)\boldsymbol{s}_2\\ \left(\boldsymbol{r}_O+\exp\left(\theta_a\tilde{\boldsymbol{s}}_a\right)\left(\boldsymbol{r}_2-\boldsymbol{r}_O\right)\right)\times\left(\exp\left(\theta_a\tilde{\boldsymbol{s}}_a\right)\boldsymbol{s}_2\right)\end{pmatrix}\triangle t_1\begin{pmatrix}\boldsymbol{0}\\ \exp\left(\theta_a\tilde{\boldsymbol{s}}_a\right)\boldsymbol{s}_1\end{pmatrix}\end{array}\right\} \tag{30}$$

$\left\{\boldsymbol{S}_{f,\mathrm{SPM}}\right\}\subseteq\left\{\boldsymbol{S}'_{f,l_{\mathrm{VI}}}\right\}_{\mathrm{PR}}$ lets $\mathrm{R}_a\mathrm{P}_1\underline{\underline{\mathrm{R}}}_2\mathrm{R}_b\mathrm{R}_c$ to be a feasible limb structure of SPMs.

Using this manner, six four-DOF and thirty-seven five-DOF derivative limb structures can be synthesized. In these five-DOF derivative limbs, when three adjacent joints generated three-DOF planar motions (two translations in a plane and one rotation perpendicular to that plane), the R joints in these three joints can be adjusted to not pass point *O*, twenty-one additional derivative limb structures can be obtained in this way. Furthermore, when two $\underline{\underline{\mathrm{R}}}$ joints and their adjacent one R joint intersects at a common point *A*, this R joint can be adjusted to not pass point *O*, resulting in three additional derivative limb structures. Consequently, totally one three-DOF, eight four-DOF and sixty-four five-DOF feasible limb structures of SPMs are synthesized as shown in Table 1.

According to Eq. (14), the assembly conditions for SPMs can be concluded through deriving the requirements that the one or two-DOF translations generated by the limbs in a SPM have no intersection:

(1) The spherical centers of all limbs should be placed to be coincident.

(2) When all limbs can generate translations with fixed directions, at least one of the following conditions should be satisfied.

(a) P joints in limbs generating one-DOF translations with fixed directions cannot be parallel to each other.

(b) Each translational plane that the two-DOF translations generated by a limb are parallel to cannot have common vertical planes.

Table 1. Feasible limb structures of SPMs.

Standard limb structures	Derivative limb structures
$R_aR_bR_c$	
$P_1R_aR_bR_c$	$R_aP_1R_bR_c$, $R_aR_bP_1R_c$, $R_aR_bR_cP_1$
$\underline{\underline{R}}_1R_aR_bR_c$	$R_a\underline{\underline{R}}_1R_bR_c$, $R_aR_b\underline{\underline{R}}_1R_c$, $R_aR_bR_c\underline{\underline{R}}_1$
$P_1P_2R_aR_bR_c$	$R_aP_1P_2R_bR_c$, $R_aR_bP_1P_2R_c$, $R_aR_bR_cP_1P_2$, $P_1R_aP_2R_bR_c$, $P_1R_aR_bP_2R_c$, $P_1R_aR_bR_cP_2$, $R_aP_1R_bP_2R_c$, $R_aP_1R_bR_cP_2$, $R_aR_bP_1R_cP_2$, $P_1P_2\underline{\underline{R}}_aR_bR_c$, $P_1\underline{\underline{R}}_aP_2R_bR_c$, $\underline{\underline{R}}_aP_1P_2R_bR_c$, $R_aP_1P_2\underline{\underline{R}}_bR_c$, $R_aP_1\underline{\underline{R}}_bP_2R_c$, $R_a\underline{\underline{R}}_bP_1P_2R_c$, $R_aR_bP_1P_2\underline{\underline{R}}_c$, $R_aR_bP_1\underline{\underline{R}}_cP_2$, $R_aR_b\underline{\underline{R}}_cP_1P_2$
$\underline{\underline{R}}_1\underline{\underline{R}}_2R_aR_bR_c$	$R_a\underline{\underline{R}}_1\underline{\underline{R}}_2R_bR_c$, $R_aR_b\underline{\underline{R}}_1\underline{\underline{R}}_2R_c$, $R_aR_bR_c\underline{\underline{R}}_1\underline{\underline{R}}_2$, $\underline{\underline{R}}_1R_a\underline{\underline{R}}_2R_bR_c$, $\underline{\underline{R}}_1R_aR_b\underline{\underline{R}}_2R_c$, $\underline{\underline{R}}_1R_aR_bR_c\underline{\underline{R}}_2$, $R_a\underline{\underline{R}}_1R_b\underline{\underline{R}}_2R_c$, $R_a\underline{\underline{R}}_1R_bR_c\underline{\underline{R}}_2$, $R_aR_b\underline{\underline{R}}_1R_c\underline{\underline{R}}_2$, $\underline{\underline{R}}_a\underline{\underline{R}}_a\underline{\underline{R}}_aR_bR_c$, $R_a\underline{\underline{R}}_b\underline{\underline{R}}_b\underline{\underline{R}}_bR_c$, $R_aR_b\underline{\underline{R}}_c\underline{\underline{R}}_c\underline{\underline{R}}_c$, $\underline{\underline{R}}_1\underline{\underline{R}}_2\underline{\underline{R}}_aR_bR_c$, $R_a\underline{\underline{R}}_1\underline{\underline{R}}_2\underline{\underline{R}}_bR_c$, $R_aR_b\underline{\underline{R}}_1\underline{\underline{R}}_2\underline{\underline{R}}_c$
$P_1\underline{\underline{R}}_2R_aR_bR_c$	$R_aP_1\underline{\underline{R}}_2R_bR_c$, $R_aR_bP_1\underline{\underline{R}}_2R_c$, $R_aR_bR_cP_1\underline{\underline{R}}_2$, $P_1R_a\underline{\underline{R}}_2R_bR_c$, $P_1R_aR_b\underline{\underline{R}}_2R_c$, $P_1R_aR_bR_c\underline{\underline{R}}_2$, $R_aP_1R_b\underline{\underline{R}}_2R_c$, $R_aP_1R_bR_c\underline{\underline{R}}_2$, $R_aR_bP_1R_c\underline{\underline{R}}_2$, $\underline{\underline{R}}_2P_1R_aR_bR_c$, $R_a\underline{\underline{R}}_2P_1R_bR_c$, $R_aR_b\underline{\underline{R}}_2P_1R_c$, $R_aR_bR_c\underline{\underline{R}}_2P_1$, $\underline{\underline{R}}_2R_aP_1R_bR_c$, $\underline{\underline{R}}_2R_aR_bP_1R_c$, $\underline{\underline{R}}_2R_aR_bR_cP_1$, $R_a\underline{\underline{R}}_2R_bP_1R_c$, $R_a\underline{\underline{R}}_2R_bR_cP_1$, $R_aR_b\underline{\underline{R}}_2R_cP_1$, $P_1\underline{\underline{R}}_a\underline{\underline{R}}_aR_bR_c$, $\underline{\underline{R}}_aP_1\underline{\underline{R}}_aR_bR_c$, $\underline{\underline{R}}_a\underline{\underline{R}}_aP_1R_bR_c$, $R_aP_1\underline{\underline{R}}_b\underline{\underline{R}}_bR_c$, $R_a\underline{\underline{R}}_bP_1\underline{\underline{R}}_bR_c$, $R_a\underline{\underline{R}}_b\underline{\underline{R}}_bP_1R_c$, $R_aR_bP_1\underline{\underline{R}}_c\underline{\underline{R}}_c$, $R_aR_b\underline{\underline{R}}_cP_1\underline{\underline{R}}_c$, $R_aR_b\underline{\underline{R}}_c\underline{\underline{R}}_cP_1$

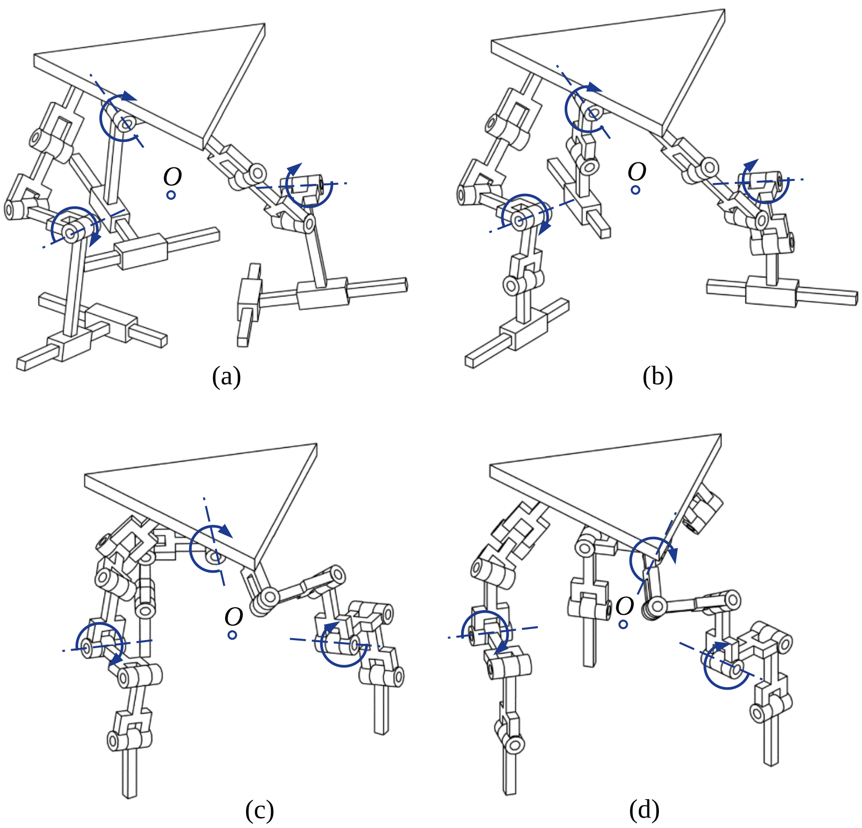

Fig. 6. Typical SPMs with symmetrical structures: (a) 3-$P_1P_2R_aR_bR_c$; (b) 3-$P_1\underline{\underline{R}}_aR_aR_bR_c$; (c) 3-$\underline{\underline{R}}_a\underline{\underline{R}}_aR_aR_bR_c$; (d) 3-$\underline{\underline{R}}_1\underline{\underline{R}}_aR_aR_bR_c$.

(c) There exists one P joint in a limb which is not parallel to the translational plane of another limb.
(3) When one or more limbs only generate translations along circles, at least one of the following conditions should be satisfied.
(a) At least one limb can generate translation with fixed direction.
(b) R joints with coincident axes which do not pass through the spherical center cannot exist in all limbs.
(c) If all limbs generate two translations along circles, there should be more than three limbs satisfying condition (3-b).

Based upon these assembly conditions, any SPM can be synthesized using the feasible limbs. Three typical SPMs with symmetrical structures are given in Fig. 6. The R_a joints can be selected as actuation joints for all these symmetrical SPMs.

4.2 Kinematic analysis of SPMs

According to the derivations in Section 2, parametric model of a SPM can be directly obtained by differentiating its topological model. Topological model of 3-$P_1P_2R_aR_bR_c$ in Fig. 7 can be formulated as

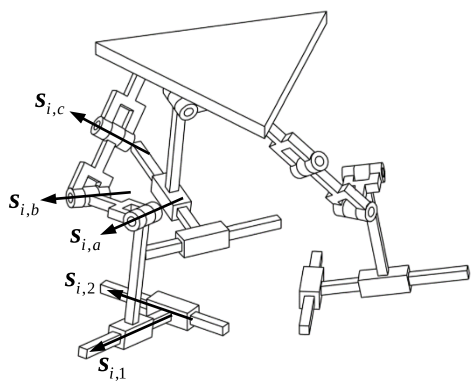

Fig. 7. 3-$P_1P_2R_aR_bR_c$ SPM.

$$\left\{\boldsymbol{S}_{f,i}\right\}=\left\{2\tan\frac{\theta_{i,c}}{2}\begin{pmatrix}\boldsymbol{s}_{i,c}\\ \boldsymbol{r}_O\times\boldsymbol{s}_{i,c}\end{pmatrix}\triangle 2\tan\frac{\theta_{i,b}}{2}\begin{pmatrix}\boldsymbol{s}_{i,b}\\ \boldsymbol{r}_O\times\boldsymbol{s}_{i,b}\end{pmatrix}\triangle 2\tan\frac{\theta_{i,a}}{2}\begin{pmatrix}\boldsymbol{s}_{i,a}\\ \boldsymbol{r}_O\times\boldsymbol{s}_{i,a}\end{pmatrix}\triangle t_{i,2}\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_{i,2}\end{pmatrix}\triangle t_{i,1}\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{s}_{i,1}\end{pmatrix}\right\} \tag{31}$$

$$\left\{\boldsymbol{S}_{f,\mathrm{SPM}}\right\}=\left\{\boldsymbol{S}_{f,1}\right\}\cap\left\{\boldsymbol{S}_{f,2}\right\}\cap\left\{\boldsymbol{S}_{f,3}\right\}=\left\{2\tan\frac{\theta_c}{2}\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_O\times\boldsymbol{s}_c\end{pmatrix}\triangle 2\tan\frac{\theta_b}{2}\begin{pmatrix}\boldsymbol{s}_b\\ \boldsymbol{r}_O\times\boldsymbol{s}_b\end{pmatrix}\triangle 2\tan\frac{\theta_a}{2}\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_O\times\boldsymbol{s}_a\end{pmatrix}\right\} \tag{32}$$

According to Eq. (12) and Eq. (18), the parametric model of this SPM can be directly formulated as

$$\left\{\boldsymbol{S}_{t,\mathrm{SPM}}\right\}=\left\{\dot{\boldsymbol{S}}_{f,\mathrm{SPM}}\Big|_{\substack{\theta_{i,j}=0,\\ t_{i,k}=0}\ \substack{i=1,2,3\\ j=a,b,c\\ k=1,2}}\right\}=\mathrm{span}\left\{\omega_c\begin{pmatrix}\boldsymbol{s}_c\\ \boldsymbol{r}_O\times\boldsymbol{s}_c\end{pmatrix},\ \omega_b\begin{pmatrix}\boldsymbol{s}_b\\ \boldsymbol{r}_O\times\boldsymbol{s}_b\end{pmatrix},\ \omega_a\begin{pmatrix}\boldsymbol{s}_a\\ \boldsymbol{r}_O\times\boldsymbol{s}_a\end{pmatrix}\right\} \tag{33}$$

Thus, the Jacobian matrix of this SPM can be obtained which is ready for velocity, acceleration, stiffness and dynamic analysis.

$$\boldsymbol{S}_{t,\mathrm{SPM}}=\begin{bmatrix}\boldsymbol{s}_a & \boldsymbol{s}_b & \boldsymbol{s}_c\\ \boldsymbol{r}_O\times\boldsymbol{s}_a & \boldsymbol{r}_O\times\boldsymbol{s}_b & \boldsymbol{r}_O\times\boldsymbol{s}_c\end{bmatrix}\begin{bmatrix}\omega_a\\ \omega_b\\ \omega_c\end{bmatrix} \tag{34}$$

5 CONCLUSIONS

A screw theory based method to unify type synthesis in finite motion level and kinematic analysis in instantaneous level of PMs is proposed in this paper. The following conclusions are drawn.

(1) The topological and parametric models of a PM, its limbs and joints are systematically formulated employing finite and instantaneous screws. The relationship between these two models is clearly revealed using the derivative mapping between the two kinds of screws.

(2) A general and consistent process based upon algebraic and analytical screw operations to do type synthesis and kinematic analysis of PMs is given using these models.

(3) The validity of the proposed method is verified through taking SPMs as an example.

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