# ENERGY HARVESTING USING A NONLINEAR VIBRATION ABSORBER

Yu Zhang, Riccardo De Rosa, Jingyi Zhang, Mariam Alameri, Kefu Liu*
*Department of Mechanical Engineering, Lakehead University, Thunder Bay, Ontario, Canada*
*Corresponding author: *E-mail: kliu@lakeheadu.ca*

## ABSTRACT
In this study, an energy harvesting device based on a nonlinear vibration absorber is proposed to achieve two objectives: vibration suppression and energy harvesting in a wideband manner. The proposed design is described. The system modeling is addressed. The parameter characterization is presented. The performance of the nonlinear vibration absorber is tested by sweeping harmonic excitation. The testing results have shown that the device can suppress vibration and convert vibration energy into electric energy in a broadband manner.

**Keywords:** energy harvesting; nonlinear vibration absorber; vibration suppression.

# RÉCUPÉRATION D'ÉNERGIE EN UTILISANT L'AMORTISSEUR DE VIBRATIONS NON LINÉAIRE

## RÉSUMÉ
Dans cette étude, un dispositif de récupération d'énergie sur la base d'un amortisseur de vibrations non linéaire est proposé pour atteindre deux objectifs: la suppression des vibrations et de récolte d'énergie d'une manière large bande. Le design proposé est décrit. La modélisation du système est adressée. Le paramètre caractérisation est présenté. Les performances de l'absorbeur de vibrations non linéaire est testé par balayage excitation harmonique. Les résultats des tests ont montré que le dispositif peut supprimer les vibrations et convertir l'énergie vibratoire en énergie électrique d'une manière à haut débit.

**Mots-clés :** récupération d'énergie; absorbeur de vibrations non linéaire; suppression des vibrations.remier mot-clé; deuxième mot-clé; troisième mot-clé.

# 1 INTRODUCTION

Vibration absorber is a passive device for suppression of vibration. A traditional vibration absorber consists of mass and spring [1]. When attached to a primary system that is subjected to a harmonic excitation, the vibration absorber can eliminate steady state response of the primary system at the tuning frequency. However, the combined system has two degrees of freedom, thus two potential resonances. To avoid resonance, linear vibration absorbers are effective only over a very narrow band of excitation frequencies. By adding a damper in parallel with the absorber spring, the operating bandwidth can be increased while the performance at the tuning frequency is compromised.

To overcome the narrowband problem of linear vibration absorbers, nonlinear vibration absorbers have been investigated for decades [2-5]. Most of these studies have shown that a nonlinear vibration absorber can increase operating bandwidth. The studies have also revealed that frequency-energy dependence is one potential limitation of nonlinear vibration absorbers.

Energy harvesting intends to scavenge energy from ambient environments or free sources such as sun, wind, wasted heat, and vibration, etc. Harvested energy is especially useful in the area of wireless or remote networks [6], where getting power to sensor networks would be difficult or prohibitive. Vibration energy harvesters use the principles of a spring-mass-damper system subjected to a base excitation. Transduction methods such as electromagnetic inductance and piezoelectric phenomenon are employed to convert mechanical energy to electrical energy.

The linear spring-mass-damper energy harvester is most efficient when it resonates with base excitation. In other words, its effectiveness is limited to a small region of frequencies. To address this problem nonlinear energy harvesters have been studied in order to gain broadband energy harvesting [7-10]. In this study, a nonlinear vibration absorber is proposed to achieve both of the aforementioned objectives: wideband vibration suppression and broadband energy harvesting. The nonlinear stiffness spring used in the proposed absorber consists of two pairs of permanent magnets. Each of pairs is formed by a magnet attached to the absorber mass and a magnet fixed to the absorber base. The polarities of the two magnets are arranged so that they repel each other. The degree of nonlinearity can be varied by adjusting the gap distance between the two fixed magnets.

The rest of the paper is organized as follows. Section 2 describes the design and modeling of the nonlinear vibration absorber. Section 3 presents characterization of the absorber system. Section 4 focuses on testing. Section 5 draws the main conclusions of the study.

# 2 NONLINEAR VIBRATION ABSORBER

Figure 1 shows the developed nonlinear vibration absorber. The absorber mass consists of an aluminum block (1), two permanent magnets and their holders (2), and two linear bearings (3). The absorber mass is supported by two precision rods (4) through the linear bearings so that it can translate freely. The two permanent magnets (5) are fixed in the absorber base (6). A coil (7) is mounted onto one of the fixed magnets. The inner diameter of the coil is slightly bigger than the oscillating magnet to allow it to move freely in and out. All of the four magnets are 1 inch sintered Neodymium, grade 40 with nickel coating.

Figure 2 illustrates the polarity arrangements of the four magnets. Each pair of the oscillating and fixed magnets is in a repelling mode such that it forms a magnetic spring. As shown in the figure $d_1$ and $d_2$ denotes the distances between the oscillating magnets and fixed magnets, respectively. If $D$ is the gap distance between the fixed magnets, $h$ the thickness of the absorber mass block, and $x_a$ the displacement of the absorber mass, $d_1$ and $d_2$ are calculated by

$$d_1 = \frac{D-h}{2} - x_a \tag{1}$$

and

$$d_2 = \frac{D-h}{2} + x_a \tag{2}$$

The repelling forces between the oscillating magnets and fixed magnets are defined by [11]

$$F_1 = \frac{a_1}{(d_1 + a_2)^4} \tag{3}$$

and

$$F_2 = \frac{a_1}{(d_2 + a_2)^4} \tag{4}$$

where $a_1$ and $a_2$ are the constants to be determined.

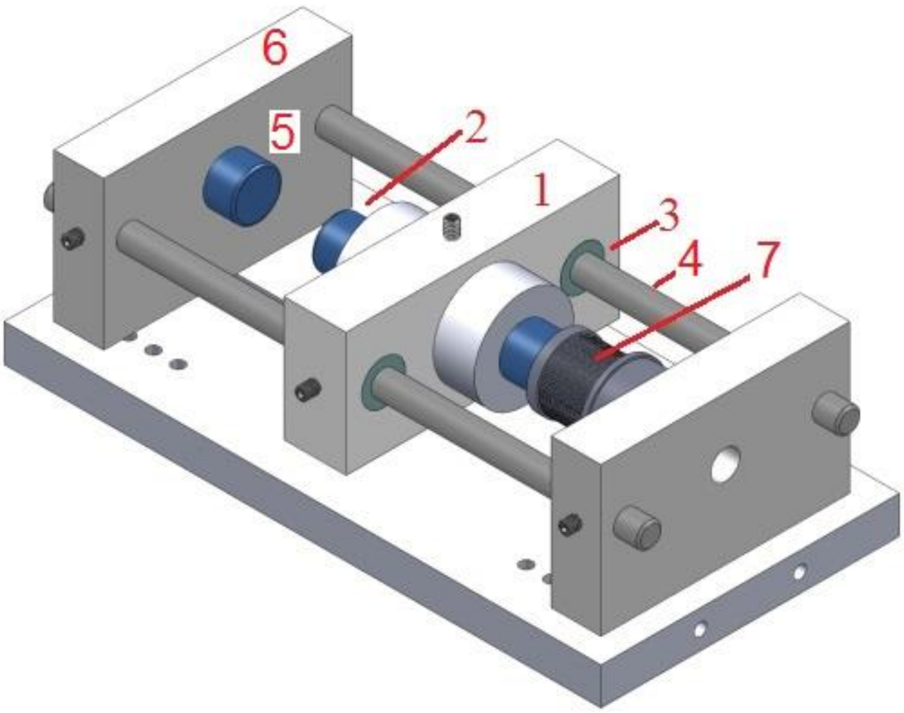


Fig. 1. Nonlinear vibration absorber

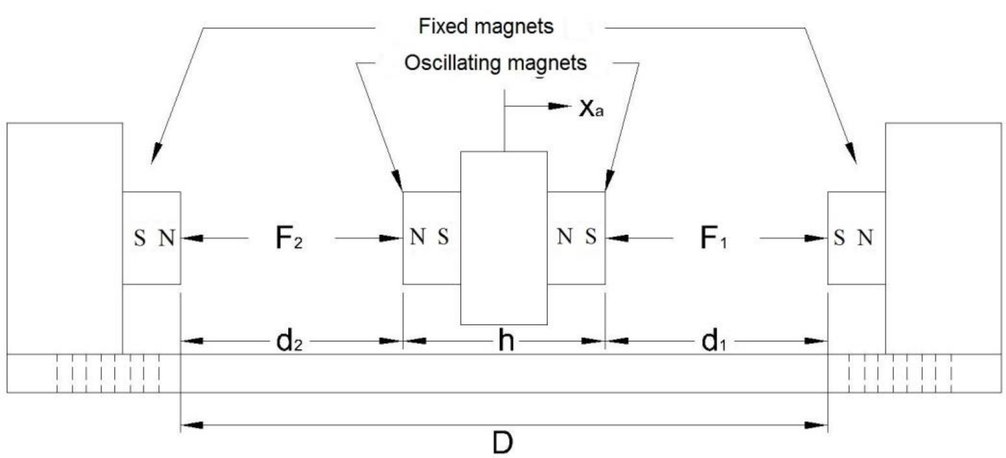


Fig. 2. Schematic of illustrating the absorber spring.

If $x_a = 0$ or the absorber mass's displacement is zero, $F_1 = F_2$. If $x_a$ is not zero, the net force on the absorber mass can be expressed as the following equation:

$$\Delta F = F_1 - F_2 = \frac{a_1}{(q - x_a)^4} - \frac{a_1}{(q + x_a)^4} = 8a_1 q \frac{x_a (q^2 + x_a{}^2)}{(q^2 - x_a{}^2)^4} \tag{5}$$

where

$$q = \frac{D - h}{2} + a_2 \tag{6}$$

Differentiating $\Delta F$ with respect to $x_a$ gives the stiffness of the absorber spring.

$$k_a = \frac{d\Delta F}{dx_a} = 8a_1 q \frac{5x_a{}^4 + 10q^2 x_a{}^2 + q^4}{(q^2 - x_a{}^2)^5} \tag{7}$$

## 3 CHARACTERIZATION OF THE NONLINEAR VIBRATION ABSORBER

The absorber mass is easily found to be $m_a = 1.186$ kg. To determine the absorber stiffness defined by Eq. (7), the constants $a_1$ and $a_2$ need to be found. Figure 3 shows a setup used to measure the repelling force between the two magnets. The fixed magnet is mounted on a force sensor that is fastened to the absorber base. By moving the oscillating magnet, the forces corresponding to various distances can be measured. The dots in Fig. 4 represent the measured values. Using the measured values to curve-fit Eq. (3) yields the constants $a_1 = 1.395 \times 10^{-4}$ N.m$^4$ and $a_2 = 2.7410 \times 10^{-2}$ m. The solid line in Fig. 4 give the best curve fit.

Now by specifying $D$, the relationship between the restoring force of the absorber spring and the displacement of the absorber mass can be obtained by Eq. (5). Figure 5 shows such the relationships for three $D$ values. It can be seen that the restoring forces show a typical behavior of a hardening spring. The smaller the gap distance $D$, the more hardening the spring becomes.

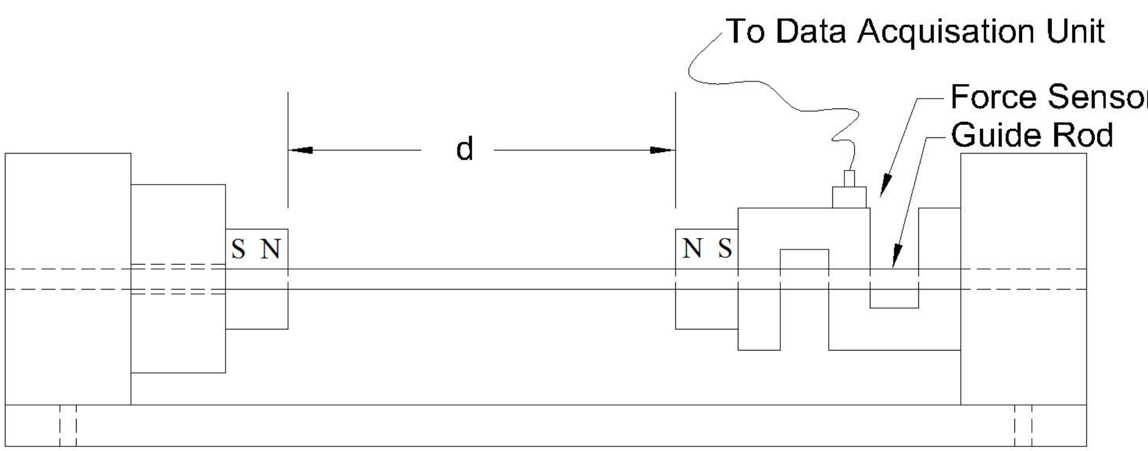


Fig. 3. Setup to measure the force between the two magnets.

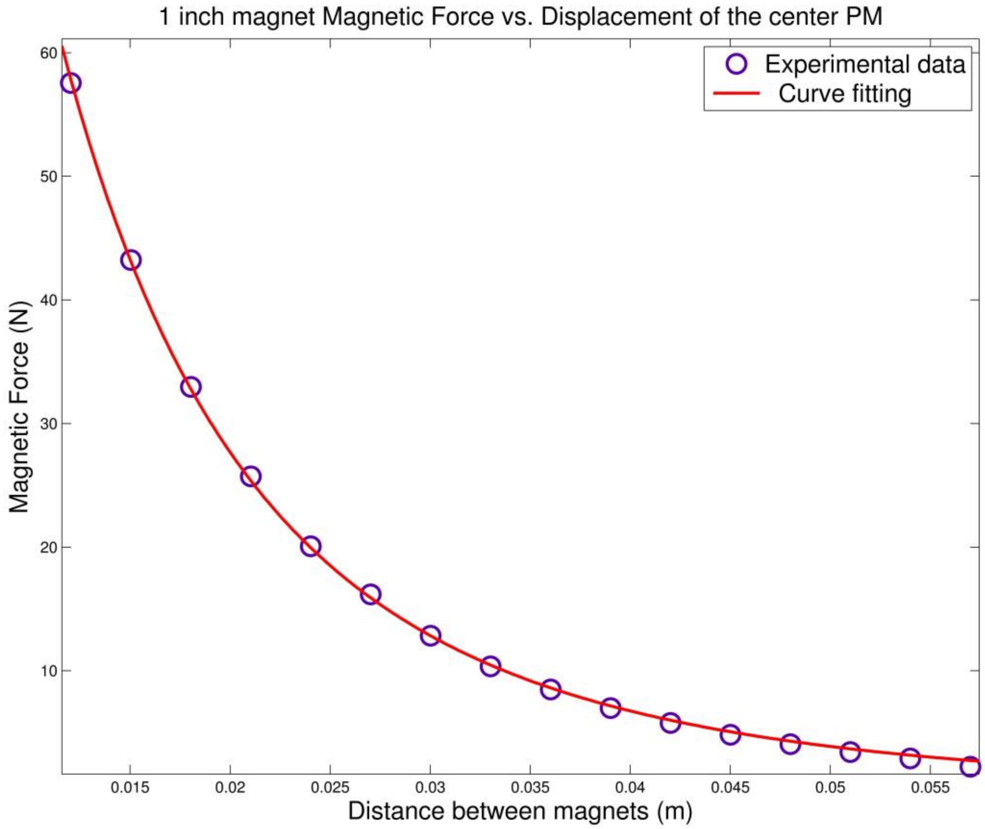


Fig. 4. Magnetic force vs. distance between the two magnets.

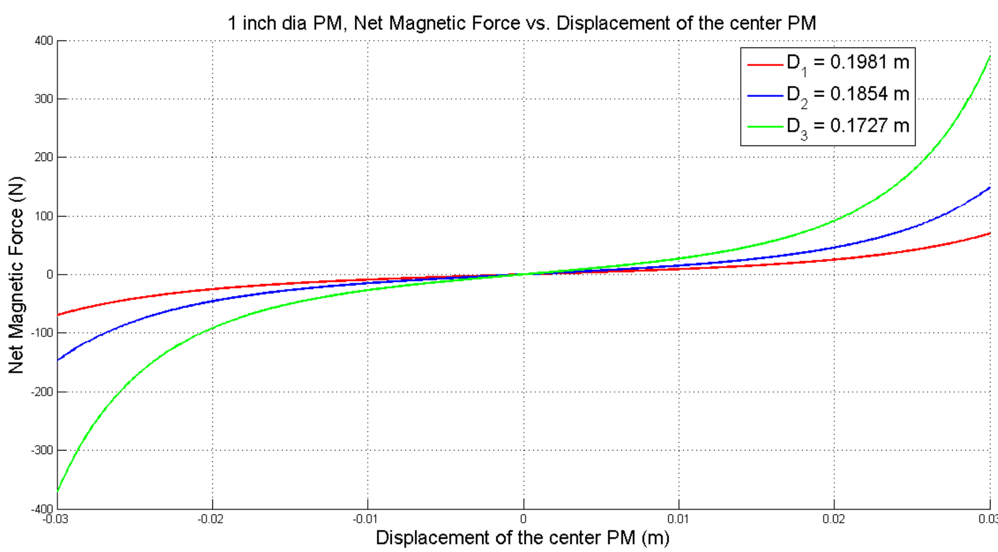


Fig. 5. Restoring force of the absorber spring vs the displacement of the absorber mass.

The restoring force or Eq. (5) can be approximated by a cubic polynomial:

$$F = k_1 x + k_3 x^3 \tag{8}$$

where $k_1$ is the linear stiffness and $k_3$ is the nonlinear stiffness. By curve-fitting the restoring forces given in Fig. 5, the values of $k_1$ and $k_3$ can be found and are summarized in Table 1. Figure 6 shows the stiffness curves defined by Eq. (8). Table 1 also lists the natural frequencies computed using $f_a = \sqrt{k_1/m_a} \big/ 2\pi$ and those identified using free responses.

Table 1. Absorber stiffness

| $D$ (m) | $k_1$ (N/m) | $k_3$ (N/m$^3$) | Computed $f_a$ (Hz) | Identified $f_a$ (Hz) |
|---|---|---|---|---|
| $D_1 = 0.19812$ | 800.4 | $9.461\times10^5$ | 4.134 | 4.208 |
| $D_2 = 0.18542$ | 1311 | $1.921\times10^6$ | 5.291 | 5.413 |
| $D_3 = 0.17272$ | 2267 | $4.232\times10^6$ | 6.957 | 6.951 |

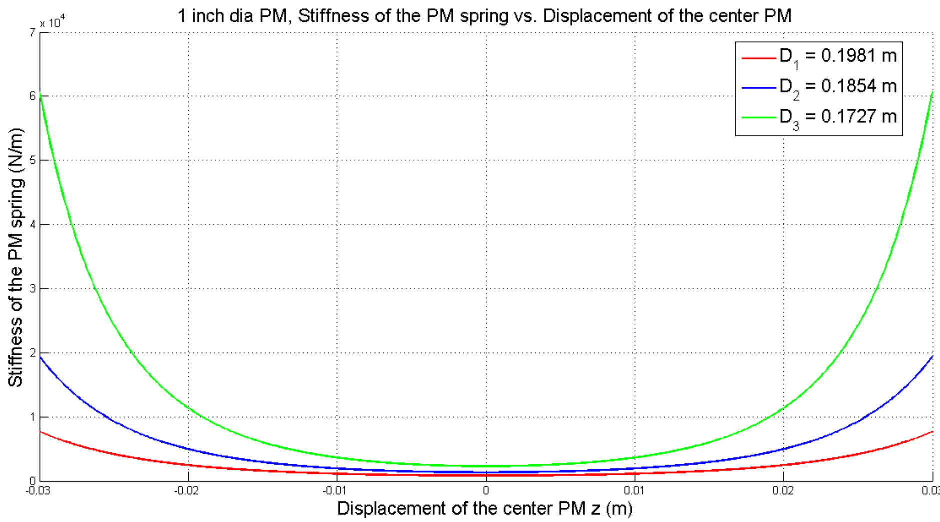


Fig. 6. Absorber stiffness vs. displacement of the absorber mass.

## 4 TESTING

Figure 7 shows the setup for testing. The absorber is fastened to a base plate that is supported by two aluminum plates. The combination of the absorber base, guide rods, and base plate forms the primary mass while the support plates become the primary spring. The primary mass is equal to $m_p = 5.328$ kg and the primary spring stiffness is equal to $k_p = 10{,}967$ N/m such that the natural frequency of the primary system is $f_p = 7.22$ Hz. To excite the primary mass, an electromagnet is used. When the harmonic current is applied to the electromagnet, the generated varying flux interacts to a small permanent magnet glued on the primary mass so that an approximate harmonic force is generated. Vibration of the primary mass is measured by an accelerometer. A PC computer equipped with B & K Pulse data acquisition system is used to convert the acceleration signal into the digital signal and convert the digital exciting signal to the analogue exciting signal for driving the electromagnet.

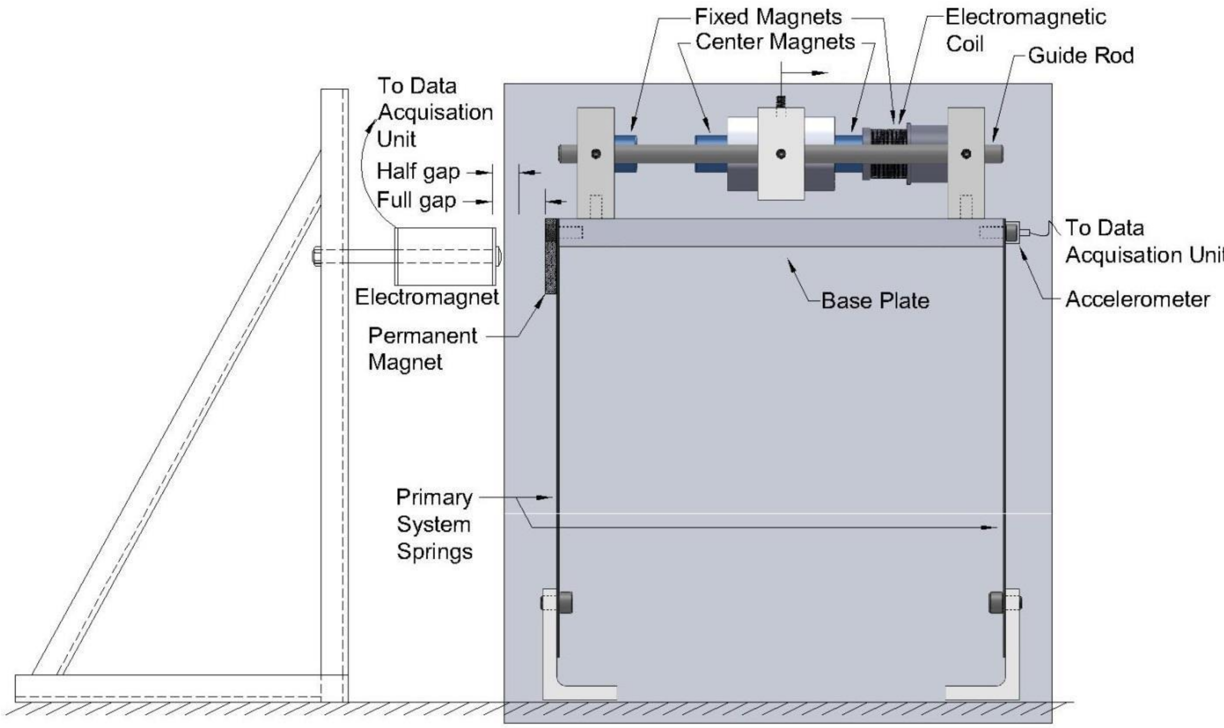


Fig. 7. Setup of the testing system.

The energy harvesting circuit consists of the coil, a variable resister used as a load resistor, and a switch. The width of the coil is about 1 inch wide, and it has 7 layers of AWG28 wire. The resistance of the coil is 15 Ohms. The resistance of the load resistor is set to be 15 Ohms as well. The circuit can be switched to open or closed manually. The coil is attached to a holder that can tightly fit to the fixed magnet. The coil is positioned such that the head of the oscillating magnet is in the middle of the coil so to maximize the power generation. Figure 8 shows the coil attachment.

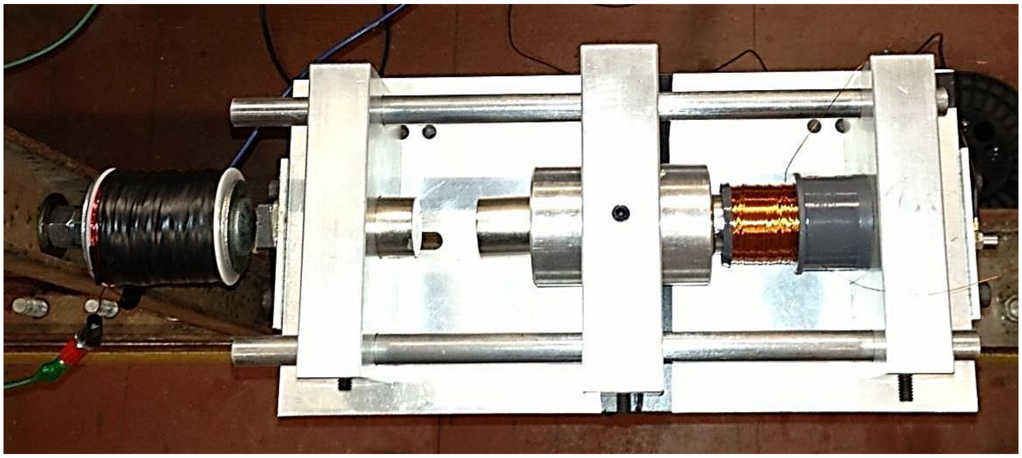

Fig. 8. Photo of the testing setup.

The performance of energy harvesting is tested by sweeping excitation. The electromagnet is driven by a harmonically varying current provided by a power amplifier which receives a harmonic voltage signal generated by Pulse system. The steady state response is recorded. The process is repeated by increasing the exciting frequency 0.25 Hz. As shown in Table 1, the natural frequency with the gap distance of $D_3$ is closest to the natural frequency of the primary system. Therefore, the nonlinear vibration absorber with $D_3$ is strongly coupled with the primary system. Thus, only the results with the setup of $D_3$ is reported here. Figure 9 shows the root mean square (RMS) value of the acceleration signal vs. the exciting frequency. It can be seen that damping due to energy harvesting is beneficial at suppression of the amplitude at resonances. Figure 10 compares the coil output voltages for open circuit and closed circuit. Apparently the energy harvester is most effective at resonances. Table 2 lists the output voltages and calculated powers using $P = V^2/R$.

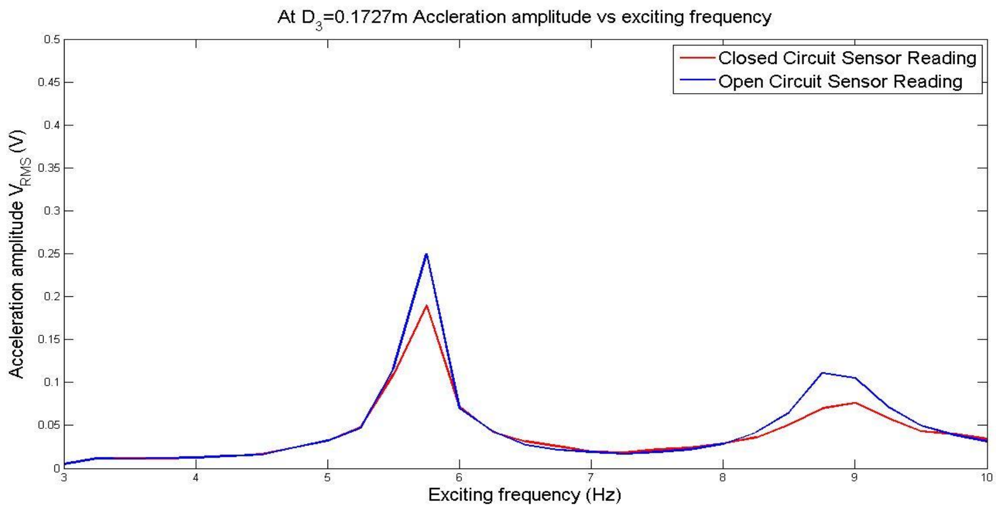


Fig. 9. Root mean square (RMS) value of acceleration signal vs. the exciting frequency.

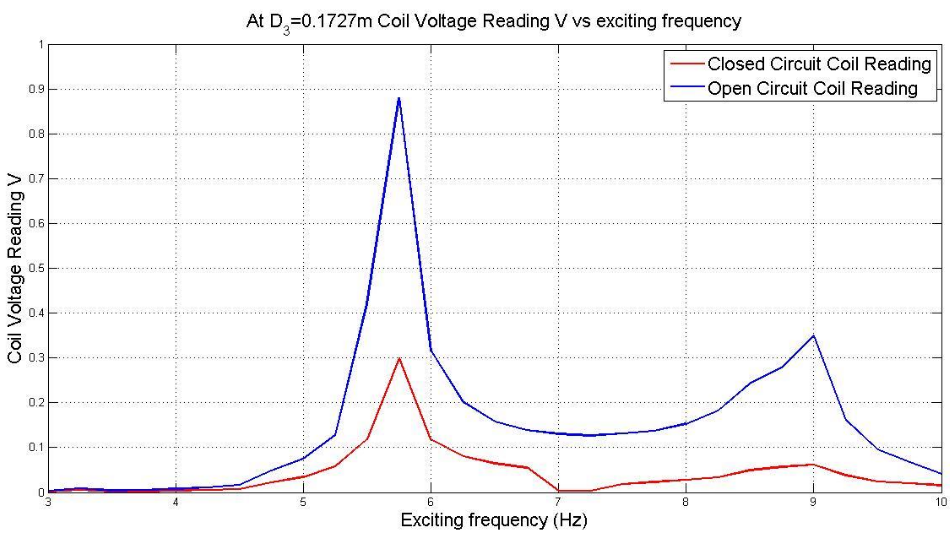


Fig. 10. Coil closed and open circuit reading.

Table 2. Coil voltage and power generation

| Exciting frequency $f$ (Hz) | Open circuit | | Closed circuit | |
|---|---|---|---|---|
| | Voltage (V) | Power (mW) | Voltage (V) | Power (mW) |
| 5.75 | 0.880 | -- | 0.299 | 5.96 |
| 9.00 | 0.349 | -- | 0.0613 | 0.251 |
| 7.25 | 0.127 | -- | 0.00294 | 0.000576 |

# 5 CONCLUSIONS

An energy harvesting device based on a nonlinear vibration absorber has been developed. Two pairs of oscillating and fixed permanent magnets have been used to form a nonlinear absorber spring. The vibration absorber has been designed so that it has a minimum mechanical damping. The nonlinear stiffness model has been developed. It is shown that the nonlinearity of the absorber spring can be varied by adjusting the gap distance of the two fixed magnets. The experimental identification has been used to determine the system parameters. By attaching the device to a primary system, seeping harmonic excitation has been conducted to test performance. The testing results have shown that the device can realize two intended objectives: vibration suppression and energy harvesting in a broadband manner.

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