SYMBOLIC FORMULATION OF PATH- AND SURFACE-FOLLOWING JOINTS FOR MULTIBODY DYNAMICS

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ABSTRACT

We present a specialized Single degree-of-freedom (DOF) Equivalent Kinematic (SEK) joint that constrains motion to a spatial path. The SEK joint is intended to be used for 1) the model reduction of 1-DOF systems; and 2) modelling systems with complex 1-DOF kinematics that cannot be accurately or easily represented using conventional modelling techniques. The joint is implemented in the graph-theoretic symbolic multibody modeling environment of MapleSim and is formulated in such a way that a single ordinary differential equation is used to describe the resulting kinematic pair. The joint can be extended to model compliance as well as 2-DOF motion along a surface using the Double-DOF Equivalent Kinematic (DEK) joint. Example applications of the joint are: the reduction of vehicle suspension systems, or the representation of biological joints.

Keywords: multibody dynamics; path following joint; vehicle dynamics.

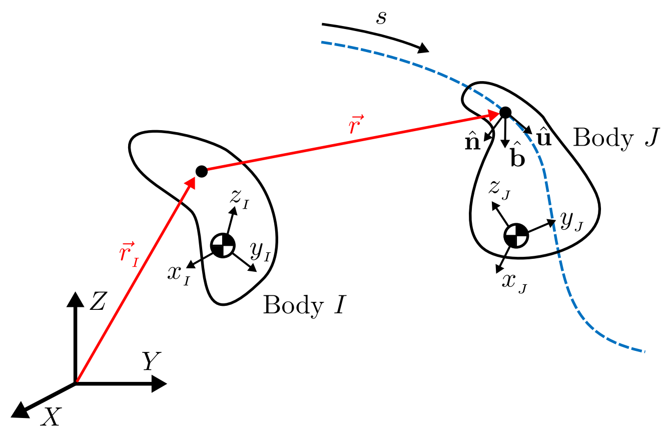

Fig. 1. General kinematics of SEK joint

1. EXTENDED ABSTRACT

We present a specialized Single degree-of-freedom (DOF) Equivalent Kinematic (SEK) joint that constrains motion to a spatial path. The SEK joint is intended to be used for 1) the model reduction of 1-DOF systems; and 2) modelling systems with complex 1-DOF kinematics that cannot be accurately or easily represented using conventional modelling techniques. The joint is implemented in the graph-theoretic symbolic multibody modeling environment of MapleSim and is formulated in such a way that a single ordinary differential equation is used to describe the resulting kinematic pair. The joint can be extended to model compliance. This adds extra mathematical complexity, but it can be a desired trade-off when modelling systems in which the compliant properties can have a significant impact on their behaviour [1, 2]. A second extension of the SEK joint is to add an additional translational DOF, so that one body is constrained to move along a surface, rather than a spatial curve, relative to another body. This results in a 2-DOF joint that is called the Double-DOF Equivalent Kinematic (DEK) joint. In this joint, 2 ODEs are used to represent the kinematic pair.

The goal of the SEK joint is to constrain one body to move along a reference path, relative to another body (Figure 1). To achieve the desired mathematical simplicity, namely representing the resulting kinematic pair with a single ODE, the definition of the reference path must be chosen correctly. The distance along the reference path, the path-length (s), is chosen as the independent coordinate for the joint. Doing so allows the components of the position vector from body I to body J ($\vec{r}$ – i.e. the reference path) and the three Euler angles (θ, ϕ, ψ) used to orient body J relative to body I to be expressed as independent functions of s:

$$\vec{r}(s) = r_x(s)\hat{\mathbf{i}} + r_y(s)\hat{\mathbf{j}} + r_z(s)\hat{\mathbf{k}} \tag{1}$$

$$\{\theta(s), \phi(s), \psi(s)\} = \{S_\theta(s), S_\phi(s), S_\psi(s)\}. \tag{2}$$

The translational and rotational motion ($\mathscr{M}$) and reaction ($\mathscr{F}$) spaces of the joint are expressed using the tangential ($\hat{\mathbf{u}}$), normal ($\hat{\mathbf{n}}$), and binormal ($\hat{\mathbf{b}}$) unit vectors that are computed using the Frenet-Serret equations [3].

$$\mathscr{M}_T = \hat{\mathbf{u}}(s) \tag{3}$$

$$\mathscr{F}_T = \langle \hat{\mathbf{n}}(s), \hat{\mathbf{b}}(s) \rangle \tag{4}$$

$$\mathscr{M}_R = \emptyset \tag{5}$$

$$\mathscr{F}_R = \langle \hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}} \rangle \tag{6}$$

To ensure the dynamics of the joint are correct, the rotational dynamics must be coupled to the translational dynamics. This is required because the joint is defined as having a single translational DOF which means that the rotational dynamics do not appear in the generated equations of motion. A torque projection force is calculated using

$$F_{TP} = \sum \vec{T}_{net} \cdot \vec{p}(s) \tag{7}$$

and is applied along the motion space ($\hat{\mathbf{u}}$) of the joint. In (7), $\vec{T}_{net}$ is the net torque in the joint and $\vec{p}(s)$ is the change in the relative orientation of the two constrained bodies with respect to s.

To introduce compliance to the SEK joint, the DOF is increased from one to six. The five additional coordinates that represent the small displacements about the specified path are: 1) n, the translational deflection in the $\hat{\mathbf{n}}$ direction; 2) b, the translational deflection in the $\hat{\mathbf{b}}$ direction; 3) θ_d, the rotational deflection about the first body-fixed Euler rotation axis; 4) ϕ_d, the rotational deflection about the second body-fixed Euler rotation axis; and 5) ψ_d, the rotational deflection about the third body-fixed Euler rotation axis.

The position vector of the SEK joint, $\vec{r}$, from (1) is renamed to $\vec{r}_{ideal}$, and n and b are incorporated into a revised joint displacement vector:

$$\vec{r} = \vec{r}_{ideal}(s) + n\hat{\mathbf{n}} + b\hat{\mathbf{b}}. \tag{8}$$

Similarly, the joint orientations from (2) are rewritten to include the new coordinates:

$$\{\theta(s), \phi(s), \psi(s)\} = \{S_\theta(s) + \theta_d, S_\phi(s) + \phi_d, S_\psi(s) + \psi_d\}. \tag{9}$$

The definition of the DEK joint is similar to the SEK joint, but two coordinates are used in the joint. The position vector is defined as:

$$\vec{r}(s_1, s_2) = r_x(s_1, s_2)\hat{\mathbf{i}} + r_y(s_1, s_2)\hat{\mathbf{j}} + r_z(s_1, s_2)\hat{\mathbf{k}} \tag{10}$$

The motion and reaction spaces are adjusted accordingly to represent the additional DOF:

$$\mathscr{M}_T = \langle \hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2 \rangle \tag{11}$$

$$\mathscr{F}_T = \hat{\mathbf{n}} \tag{12}$$

where $\hat{\mathbf{u}}_1$ and $\hat{\mathbf{u}}_2$ are two vectors tangential to the surface. The torque projection for the SEK joint is extended in a similar fashion to represent the additional DOF.

A full vehicle model can be constructed using SEK joints to represent an unsteered rear MacPherson suspension, and DEK joints to model a steered front MacPherson suspension. An equivalent high-fidelity model is created in MapleSim using conventional modeling techniques to demonstrate the accuracy of the SEK joint as well as the simulation time improvements. The two models are driven through a double lane change maneuver. The reduced model constructed using the SEK and DEK joints simulates 2.4 times faster than the high-fidelity model. For the steering input in Figure 2a it can be seen that the response between the two models is within 1%, Figure 2b.

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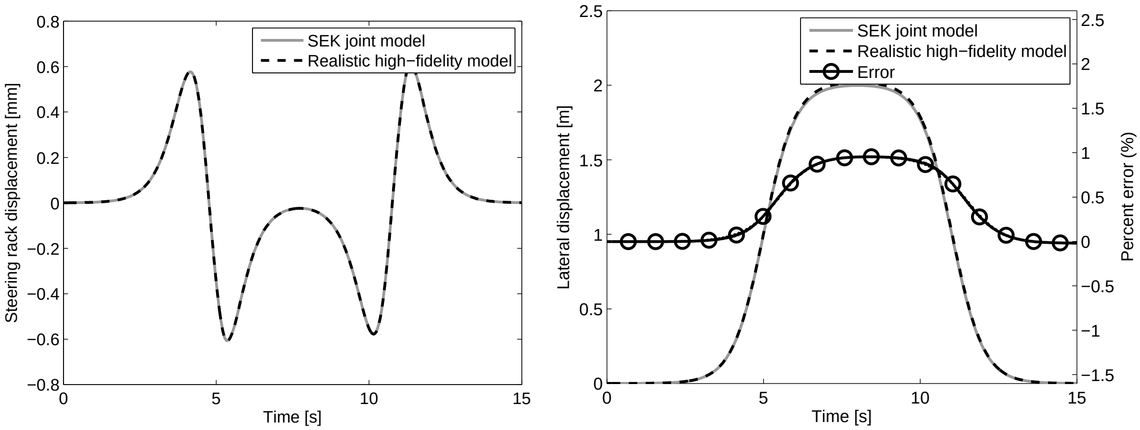

(a) Steering rack displacement versus time

(b) Vehicle lateral displacement versus time

Fig. 2. Steering rack displacement and resulting lateral displacement comparison for high-fidelity and reduced vehicle models during a double lane change maneuver

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