DIMENSIONAL SYNTHESIS OF PLANAR CABLE-DRIVEN PARALLEL ROBOTS VIA INTERVAL ANALYSIS

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ABSTRACT

This paper deals with the dimensional synthesis of the workspace of a planar cable-driven parallel robot. The dimensional synthesis problem is made equivalent to an optimization problem which is solved by resorting to interval analysis. The workspace under study, namely the wrench-feasible workspace, consists in set of poses of platform for which a predefined wrench interval can be produced by means of set of cable tensions in an admissible range. The main contribution of this paper consists in obtaining the optimum design of a cable-driven parallel robot, by having the position of one actuator, as optimization parameter. To do so, upon a convenient cost function, actuator location will be obtained for which the corresponding workspace would cover a prescribed circle. Moreover, in order to optimize the workspace, for a given wrench-feasible workspace of the robot, the maximal wrench-feasible workspace circle will be found using an interval-based algorithm.

Keywords: Cable-driven parallel robots; Interval analysis; Dimensional synthesis; Maximal wrench-feasible workspace.

SYNTHÈSE DIMENSIONNELLE DES ROBOTS PARALLÈLES PLANAIRES ENTRAÎNÉS PAR CÂBLES EN UTILISANT ANALYSE PAR INTERVALLES

RÉSUMÉ

Ce papier étudie la synthèse dimensionnelle de l'espace de travail d'un robot parallèle entraîné par câble. Le problème de synthèse dimensionnelle est équivalent à un problème d' optimisation qui est résolu par le recours à l'analyse par intervalle. L'espace de travail en question est l'espace des torseur-réalisable qui consiste dans la série de poses de plate-forme pour laquelle un intervalle prédéfini peut être produit au moyen d'ensemble de tensions de câble dans une plage admissible. La principale contributaion de ce papier consiste à obtenir la conception optimale d'un robot parallèl entraîné par câble en considérant la position d'un actionneur comme étant la variable d'optimisation. En effet, pour une fonction d'objective d'optimisation, l'mplacement de l'actionneur sera obtenu en tell sorte que l'espace de travail correspondant couvriraitun cercle prescrit. Par ailleurs, afin d'optimiser l'espace de travail, pour un espace de travail torseur-réalisable compute tenu du robot, le cercle de l'espace de travail torseur-réalisable maximal sera trouvée en utilisant un algorithme par intervalle.

Mots-clés : Robots Parallèles Entraînés par Câbles, L'analyse par Intervalle, Synthèse Dimensionnelle, Espace de Travail Maximale du Torseur-Réalisable.

1. INTRODUCTION

Most of conventional industrial robots have anthropomorphic characteristics and are referred to as serial robotic manipulator. These type of robots compromise some rigid links connected by prismatic or revolute joints. Serial robots have some properties of interest, such as, extended workspace, flexibility and manoeuvrability. However, they have low accuracy compared to their counterparts, parallel robots [1]. Moreover, serial robots are not able to access high amounts of velocity or acceleration. In addition, the whole wrench, i.e., external wrench and platform mass, acted on the End-Effector (EE) transmits to the base via an open loop chain which imposes high bending forces to the first joint. Although, parallel robots should not be regarded as an ultimate remedy for serial shortcomings of serial robots, but an appropriate design of a parallel robot could alleviate some kinematic drawbacks of serial robots.

Parallel robots can access more carrying capacity [2], better accuracy and high velocity and acceleration [3]. The foregoing properties candidate them for many applications such as Gough-Stewart platform [4] for flight simulators [5], Delta robots [6] for pick and place applications and Phantom robot as a Haptic interface [7]. In some particular applications, parallel robots with rigid links have some disadvantages such as complicated kinematic and dynamic equation and limited workspace, due to the stroke of actuators.

The origin of cable-driven parallel robots dates back to 1985 where Sheridan and Lansberger [8] introduced the first type of such robots. Cable-driven parallel robots are parallel mechanisms which use flexible cables to manipulate the EE. Cables are coiled on motor drums at one end and connected to the moving platform at the other end. Since long lengths of cables can be coiled on drums, the workspace of these robots can be extended even to cover a stadium or arena, e.g., SkyCam [9]. Also, due to the lightness of cables compared with rigid links, the mechanism can exhibit high amount of velocities and accelerations for a reasonable consumption of energy.

The pose (position and orientation) of the mobile platform in a cable driven parallel robot, is performed only by changing the elongation of the cables. Since cables can only carry tension forces, it is necessary to keep cables in a tension condition. Otherwise, the control of the platform will be lost. This is the main reason for which the workspace analysis of cable-driven parallel robots requires the consideration of static and kinematic properties. The static limitation of cables in cable-driven parallel robots, i.e. the impossibility to pull, can be compensated by applying redundancy into the mechanism. Despite of many mathematical complexities, redundancy can be regarded as an advantage for cable-driven parallel robots, such as singularity avoidance.

In fact, the workspace of cable-driven parallel robots are investigated under different perspectives and different classes are introduced, such as controllable, static, dynamic and wrench feasible workspace [1, 10–14]. Controllable workspace depends on just the geometry parameters of the robot. In addition to the geometry of the robot, static and dynamic workspaces are related to the weight and acceleration of the EE, respectively [11–13]. According to the definition of these three types of workspace, the range of acceptable cable forces is not considered in any of them. But a definition for the workspace of cable robots which takes into account a range of cable forces and, moreover, the interval of EE wrenches, is introduced in [14–16]; namely the Wrench-Feasible Workspace (WFW). To this aim, an approach based on interval analysis, [17], will be employed for calculating the guaranteed set of intervals which lie in WFW.

Synthesis of a robot consists of determining the position of actuators, choosing the type of actuators, arrangement of limb structure and link length [18]. In a cable-driven parallel robot, all actuators are rotary motors, the length of cables can be regarded infinite and the type of all limbs are the same and is similar with a prismatic joint. Furthermore, the case study of this paper is a 2T cable-driven robot, T stands for translational DOF, and the EE is considered to be a point mass, therefore, all cables are attached to a common point and the attachment position of cables is not important. Hence, in this case the only important criteria to define a cable-driven parallel robot is the position of the actuators.

In the design of robotic mechanical systems, the first step is to produce a conceptual design that will meet the design specifications [19]. The synthesis, is a vital phase that should be considered before manufacturing a mechanism. According to [19], the synthesis problem consists of finding the linkage that best performs a given task. One of the branches of kinematic synthesis is *dimensional synthesis* [19] which is the problem of finding the geometric parameters of, mechanism relevant to a given task. In this paper, dimensional synthesis of a planar cable-driven parallel robot is of concern. In fact, the main problem consist in finding the optimal cable-driven parallel robot for which upon prescribing a circle its corresponding WFW covers the forgoing circle. To this end, a systematic approach based on interval analysis [17, 20–24] is proposed. As a case study, a 2-DOF planar parallel cable-driven parallel robot, with three actuators, is considered in which the approach aims at determining the optimum position of actuators in such a way that the workspace of the robot includes a given circle in a 2-dimensional Cartesian space. This is common in practice where based on the intended application and prior to designing the robot the customer prescribes a conventional geometric shape for the workspace of the robot, such as a circle. Thus, an algorithm based on interval *branch & prune* will be proposed to determine the optimum position of an actuator for which the WFW (with specified wrench interval and tension range) covers the predefined circle.

In addition, in this paper, a complementary approach is introduced to the end of obtaining the Maximal Workspace Circle (MWC) for a prescribed cable-driven parallel robot. This approach ensures that weather another circle could be obtained, either equal or bigger than the circle given by the user in the first part. Therefore, this complementary algorithm will represent the optimum workspace circle of the optimum design.

The remainder of this paper is organized as follow: By reviewing basic concepts of interval analysis in Section 2, and afterwards by choosing the wrench feasible workspace, in Section 3, interval-based calculation of cable-driven parallel robots are represented. Then the algorithm to find the optimum design is broadly mentioned in Section 4 and results are represented. Finally, an algorithm to the end of obtaining the MWC is introduced and the corresponding results are discussed in Section 5.

2. INTERVAL ANALYSIS

Discretization is a method that have been widely used for investigating the workspace of robotic mechanical systems. By using this method, the answer would be a set of nodes which lie inside the workspace. However, the method cannot judge about the poses lie between two inside nodes which is broadly a drawback of such a method. A technique that would be utilized to circumvent this shortcoming is interval-based approach. Interval analysis gives answers as a set of closed intervals for which all of points that are between lower and upper bound of these intervals are guaranteed to be the answer.

In this paper, interval analysis is used as the mathematical framework. There are many benefits relevant to use interval analysis [20] such as circumventing round-off errors [21], global optimization [23], proper workspace presentation, etc. [17, 22–24]. The main attribute of interval analysis which is used in this paper is its ability to result in a set of guaranteed boxes (interval vector) which lie inside the WFW. The proposed approach considers a range of position of actuator and then by a *branch & prune* [17] method, it converges to the desired results. Furthermore, interval analysis provides an interactive visualization in the progress of calculation which is a definite asset in 2D adn 3D representations of the workspace.

Interval analysis, is a branch of mathematics which basically works with closed intervals instead of accurate numbers. An interval $[x]$ is a set of real numbers between two bounds and can be represented by:

$$[x] = [\underline{x}, \overline{x}] = \{x \in \mathbb{R} \mid \underline{x} \leq x \leq \overline{x}\} \;\; , \; (\underline{x} \leq \overline{x}) \tag{1}$$

where $\underline{x}$ and $\overline{x}$ are lower bound and upper bound, respectively. All mathematical operations such as addition or multiplication can be implemented on intervals yielding an interval. For instance:

$$[x]+[y]=[\underline{x},\overline{x}]+[\underline{y},\overline{y}]=[\underline{x}+\underline{y},\overline{x}+\overline{y}], \tag{2}$$

$$[x]\,[y]=[\min(S)\;,\;\max(S)]\;,\quad S=\{\underline{x}\underline{y},\underline{x}\overline{y},\overline{x}\underline{y},\overline{x}\overline{y}\}. \tag{3}$$

Moreover, a functions of real numbers such as $f(x)$ can be interval evaluated from a given interval, $[x]$, which results an interval $[f]=f([x])$. For example for a monotonic function like $f(x)=x^3$:

$$[f]=f([x])=[f(\underline{x}),f(\overline{x})]=[\underline{x}^3,\overline{x}^3]. \tag{4}$$

The usage of interval analysis in calculating WFW is merged into the next section and it has been extensively investigated in [11, 14].

3. WRENCH FEASIBLE WORKSPACE

It should be noted that the concepts presented in this section are to the majority of intents and purposes the same as the one presented in [14] and just a glimpse of the concept is recalled hereafter which is primordial for the optimization and dimensional synthesis purposes of this paper.

Consider $[\mathbf{f}]$ to be an interval vector of the required wrench and $[\boldsymbol{\tau}]$ is the box of allowed cable tensions as follows:

$$[\boldsymbol{\tau}]=\{\boldsymbol{\tau}\mid\tau_i\in[\tau_{i_{\min}},\tau_{i_{\max}}]\;,\;1\le i\le m\} \tag{5}$$

where for each i, $0\le\tau_{i_{\min}}<\tau_{i_{\max}}$. It should be noted that, $\tau_{i_{\max}}$ depends on maximum reachable torque of actuator or admissible cable tension while $\tau_{i_{\min}}$ is defined to ensure that cables will not be slack and usually is selected as an amount more than zero.

Definition (WFW), [14]. WFW is the set of mobile platform poses that are wrench-feasible, i.e., for which, for any wrench $\mathbf{f}$ in $[\mathbf{f}]$, there exists a vector of cable tensions $\boldsymbol{\tau}$ in such a way that: $\mathbf{W}\boldsymbol{\tau}=\mathbf{f}$.

3.1. Interval-based Algorithm of WFW

By using interval analysis tools, a given box of EE poses can be checked to be whether inside, outside or containing a piece of boundary of the WFW contour [14]. The box of poses $[\mathbf{x}]$ yields wrench matrix $\mathbf{W}$ as an interval matrix $[\mathbf{W}]=\mathbf{W}([\mathbf{x}])$ with the fundamental property of: $\forall\,\mathbf{x}\in[\mathbf{x}],\;\mathbf{W}(\mathbf{x})\in[\mathbf{W}]$. Then, the problem of distinguishing which boxes are fully inside the WFW leads to specify the feasibility of a system of interval linear equations $[\mathbf{W}]\boldsymbol{\tau}=[\mathbf{f}],\;\boldsymbol{\tau}\in[\boldsymbol{\tau}]$ which as stated in [14] would be examined via a theorem represented by J. Rohn [25]. Again as stated in [14], for the boxes that are totally outside the WFW boundary, one can resort to consistency techniques which establish a tool to check unavailability of cables to produce a particular wrench $\mathbf{f}\in[\mathbf{f}]$ by admissible tensions in $[\boldsymbol{\tau}]$.

3.2. Determination of WFW using branch & prune algorithm

Most of interval analysis algorithms, are based on branch & prune algorithm. To the sake of describing the forgoing algorithm, first of all consider the problem with a set of unknowns as $x=\{x_1,x_2,\ldots,x_n\}$. The main part of the branch & prune algorithm is the *Bisection* procedure of boxes. Assume a stage of algorithm which deals with box $\mathsf{B}_d=\{[\underline{x_1^d},\overline{x_1^d}],\ldots,[\underline{x_n^d},\overline{x_n^d}]\}$. If this box satisfies the conditions of the main problem, is saved as answer of the problem. If there is no answer of the problem under study in this box, then the box is out of range of solution and, thus excluded from the rest of procedure. Otherwise, the box B_d should be bisected yielding two new boxes $\mathsf{B}_d^{I_1}=\{[\underline{x_1^d},\overline{x_1^d}],\ldots,[\underline{x_j^d},(\underline{x_j^d}+\overline{x_j^d})/2],\ldots,[\underline{x_n^d},\overline{x_n^d}]\}$ and $\mathsf{B}_d^{I_2}=\{[\underline{x_1^d},\overline{x_1^d}],\ldots,[(\underline{x_j^d}+\overline{x_j^d})/2,\overline{x_j^d}],\ldots,[\underline{x_n^d},\overline{x_n^d}]\}$ where the j-th component of box B_d (interval $[\underline{x_j^d},\overline{x_j^d}]$) has the largest width in the box. This procedure will stop when the processing box width is smaller than a threshold vector ε. Finally, there would be three sets of boxes:

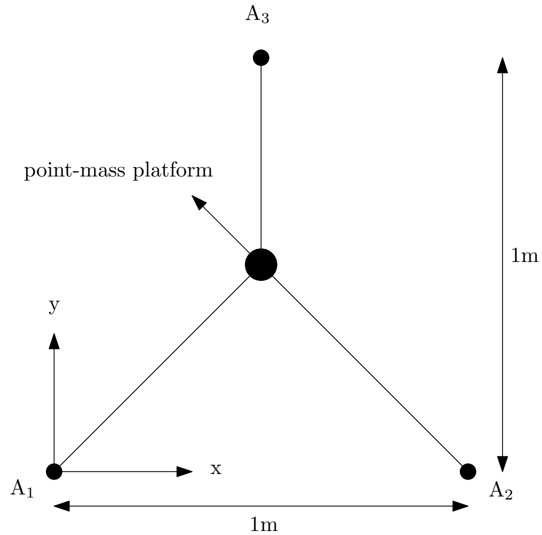

Fig. 1. Shematic of a planar 2T cable-driven robot, [26].

Algorithm 1 Optimization algorithm

Input: $A1_x$, $A2_x$, $A3_x$, $A1_y$, $A2_y$, $[A3_y]$, ε, circle(C,R) %circle(C,R) is the circle with center of C and radius of R
Output: sup$([\mathsf{B}(U)])$
$\{\mathsf{B}\} = \{[\mathsf{B}(L)], [\mathsf{B}(U)]\} \leftarrow$ Bisect$([A3_y])$
$e =$ diam(B) % Returns the widths of box
while $e > \varepsilon$ **do**
  WFW= obtain_WFW$(A1_x, A2_x, A3_x, A1_y, A2_y, [\mathsf{B}(U)], \varepsilon)$ %Returns WFW boxes regarding $[\mathsf{B}(U)]$
  if (Inside(circle(C,R),WFW)) **then** % Checks the inclusion of circle in WFW
    $\{\mathsf{B}\} =$ Bisect$([\mathsf{B}(L)])$
  else
    $\{\mathsf{B}\} =$ Bisect$([\mathsf{B}(U)])$
  end if
  $e =$ diam$([\mathsf{B}(\mathrm{U})])$ % Or $e =$ diam$([\mathsf{B}(\mathrm{L})])$
end while

1. Set of boxes which are completely inside the answer of the problem.
2. Set of boxes which are completely outside the answer of the problem.
3. Set of boxes which do not completely belong neither to answer nor to outside of the answer.

The existence of the third set is inevitable and can be shrunk (not removed) just by decreasing vector ε.

An example for the WFW of a cable-driven paralell robot, Fig. 2 will be provided in the upcoming section while introducing the optimization procedure.

4. OPTIMIZATION

Optimization is always performed with respect to a cost function. Beside economical cost functions like number of actuators which should be minimized, some other criterion like volume expansion would be considered in the case of cable-driven parallel robots [26, 27]. Reducing the volume expansion, on one

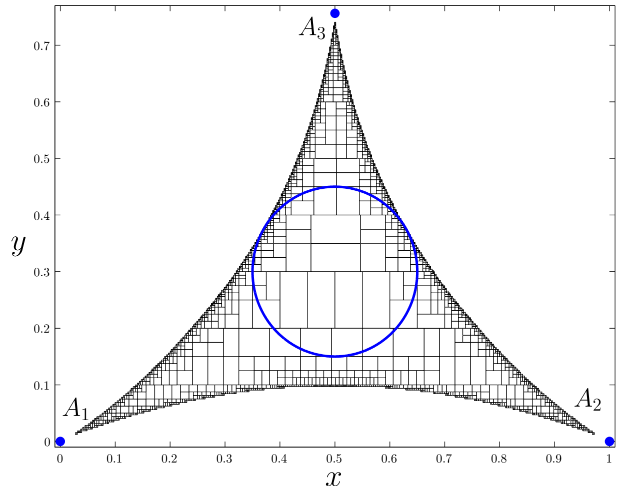

Fig. 2. Optimum solution of a 2-DOF cable-driven parallel robot for which the wrench feasible workspace covers a given circle.

hand, allows to embed robot in smaller space, which reduces costs, and on the other hand, less lengths of cables would be needed to manipulate the platform. The problem of optimization consists in: minimizing the volume expansion of the robot in such a way that the corresponding WFW embraces a predefined circle by considering the position of actuators as optimization parameters. For implementing the optimization algorithm, the position of actuators can be considered as intervals and again branch & prune algorithm will be applied.

Let assume the 2-DOF planar cable-driven parallel robot as illustrated in Fig. 1. The actuators are placed in $A_1 = (0,0)^{\mathrm{T}}$m, $A_2 = (1,0)^{\mathrm{T}}$m and $A_3 = (0.5,1)^{\mathrm{T}}$m. Assume a wrench vector of $[\mathbf{f}] = ([-20,20],[-20,20])^{\mathrm{T}}$N and admissible interval of cable tensions is $\tau_i \in [10,90]$ N. For such a problem, the WFW is computed using branch & prune algorithm with $\varepsilon = (0.001,0.001)^{\mathrm{T}}$m. wrench vector of $[\mathbf{f}] = ([-20,20],[-20,20])^{\mathrm{T}}$N and admissible interval of cable tensions is $\tau_i \in [10,90]$N. The problem consists in finding the minimum value for the y-coordinate of actuator 3, A_{3_y}, for which a circle with center of $(0.5,0.3)^{\mathrm{T}}$m and radius of 0.15m is covered by the WFW. For solving this problem, an algorithm has been developed as illustrated in Alg. 1 which is based on interval branch & prune method. For this problem, the interval $A_{3_y} = [0.6,1]$m is considered as initial search domain and threshold vector is $\varepsilon = (0.002,0.002)^{\mathrm{T}}$. In each iteration, the search range of A_{3_y} is bisected, namely the upper part ($[\mathsf{B}(U)]$) and the lower part ($[\mathsf{B}(L)]$) where the WFW is computed with respect to interval $[\mathsf{B}(U)]$ as the y-coordinate of the third actuator, A_{3_y}. If for $[\mathsf{B}(\mathsf{U})]$, the corresponding WFW contains the given circle, the design purpose is fulfilled, therefore, $[\mathsf{B}(\mathsf{U})]$ will be excluded from the rest of procedure and $[\mathsf{B}(L)]$ is considered for further operation. Otherwise, $[\mathsf{B}(U)]$ goes for bisection procedure. The next step is the bisection of the remained box and this will be pursued until the desired accuracy reaches. Finally, the result is $A_{3_y} = 0.756$m, as illustrated in Fig. 2 and the evolution of algorithm iterations is presented in Fig. 3. In Fig. 3, a third dimension is used in order to represent the evolution of y-coordinate of A_3 toward the optimal one — as indicated in Fig. 3, the optimal value is 0.756m — and the gray cylinder is representing the prescribed circle in all designs. As it can be observed, the optimum design reached where the whole circle is inside the WFW.

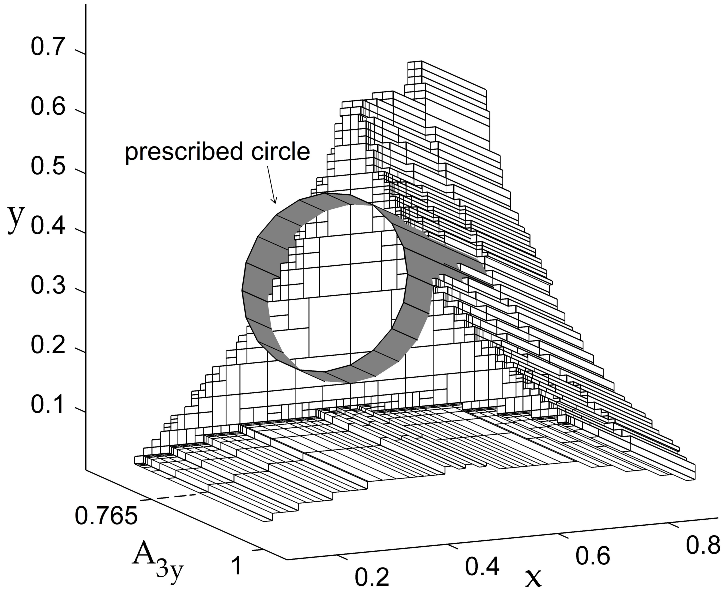

Fig. 3. Evolution of the WFW with respect to the optimization iterations.

Algorithm 2 The algorithm used for determining of the boundary boxes

Input: $\mathscr{L}_{\text{in}}$, $\mathscr{L}_{\text{neg}}, \varepsilon$
Output: $\mathscr{L}_{\text{B}}$ % The list of boundary boxes
for all $[\mathsf{B}_i] \in \mathscr{L}_{\text{neg}}$ **do**
    $A = \text{mid}([\mathsf{B}_i])$ % Returns the midpoint of box $[\mathsf{B}_i]$
    $[B_1, B_2, B_3, B_4] \leftarrow \left[A + \begin{pmatrix} \varepsilon \\ 0 \end{pmatrix}, A - \begin{pmatrix} \varepsilon \\ 0 \end{pmatrix}, A + \begin{pmatrix} 0 \\ \varepsilon \end{pmatrix}, A - \begin{pmatrix} 0 \\ \varepsilon \end{pmatrix}\right]$
    for all $B_i \in [B_1, B_2, B_3, B_4]$ **do**
        if $B_i \in \mathscr{L}_{\text{in}}$ **then**
            $[\mathsf{B}_i] \in \mathscr{L}_{\text{B}}$
            break **for**
        end if
    end for
end for

5. OBTAINING MAXIMAL CIRCLE IN THE WRENCH-FEASIBLE WORKSPACE OF CABLE-DRIVEN PARALLEL ROBOTS

The WFW of a cable-driven parallel robot is mostly consists of sharp and narrow edges close to the position of actuators, Fig. 2. These thin regions of WFW are usually undesired and quite useless in robotic applications, because of the limitation in the motion of platform. Moreover, generally, close to such boundaries the mechanism exhibits poor performances regarding some of its kinetostatic properties, such as kinematic sensitivity [19]. Therefore, for a given robot with fixed position of actuators, obtaining the Maximal Workspace Circle (MWC), which could be regarded as a conservative workspace, leads to represent the WFW as a convex and regular area. In this section, an interval-based algorithm is introduced to the end of obtaining the MWC.

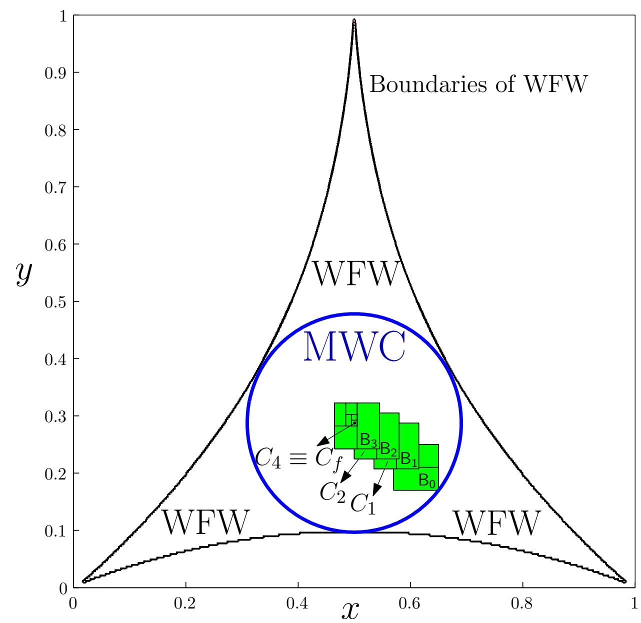

Fig. 4. The MWC obtained for WFW of the 2T robot, represented in Fig. 1.

Algorithm 2, represents a pseudo-code to the end of obtaining the set of boundary boxes of WFW. In short, this algorithm receives two sort of lists, namely, $\mathscr{L}_{\text{in}}$ and $\mathscr{L}_{\text{unc}}$ as set of WFW boxes (inside) and set of uncertain boxes which have dimension less than ε (uncertain), respectively. In fact, a branch & prune procedure start with a search space and bisects it. If any of bisected boxes fulfilled the criterion of the WFW, then it will be added to the list of inside, $\mathscr{L}_{\text{in}}$, otherwise it goes for another bisection. The procedure stops when the prescribed accuracy ε is achieved and remained boxes are added to list of uncertain boxes, $\mathscr{L}_{\text{unc}}$. For each box belonging to $\mathscr{L}_{\text{unc}}$, its midpoint is stored in A. Moreover, in line 5, four points are generated around A in such a way that keep a distance of ε from A along the main four directions. If at least one of these four points lies inside the $\mathscr{L}_{\text{in}}$, then the corresponding box is a member of set of boundary boxes, called $\mathscr{L}_B$. The mentioned procedure applies for all members of $\mathscr{L}_{\text{unc}}$ and finally a list of boundary boxes entitled $\mathscr{L}_B$ will be an input for the next algorithm.

The proposed algorithm in this paper for obtaining the MWC (for WFW) is based on a branch & prune procedure and Alg. 3 represents the corresponding pseudo-code. Two sorts of information are needed to start Alg. 3: (a) the set of boundary boxes of WFW, $\mathscr{L}_B$, obtained from Alg. 2 and (b) initial guess box, prescribed by the user. This box could be any of boxes inside WFW, most of the times the middle one is the best choice and converges more rapidly. The first step consists in bisecting the initial guess box (B_0) by the largest edge, then for the two new generated boxes, the corresponding distance to the set of boundaries, $\mathscr{L}_B$, are computed separately. The one with the greater lower bound (B_g) is kept for further operation and the other one is ignored. Then, B_g is bisected and the same procedure applies on two new generated boxes. This continues until the desired accuracy, ε, is reached. The last small box, with an acceptable approximation, could be regarded as a point, C_0. This point could be an appropriated candidate of being the center of MWC, if and only if, C_0 is not close to the edges of B_0. In other words, the distance of C_0 to edges of B_0 is greater than ε. This means that the obtained center point have still the tendency to travel through the WFW to reach the optimum center point for the MWC. Therefore the algorithm pursues the procedure by creating a new generated box (B_1) around C_0 with the same size of B_0. The procedure peruses the same computation as mentioned for generated boxes, $\mathsf{B}_1, \mathsf{B}_2, \ldots, \mathsf{B}_\mathsf{n}$, until the obtained center point is not close to the edges of

Algorithm 3 Algorithm for finding the MWC

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Input: $\mathscr{L}_B$ , $\varepsilon$ , $\mathsf{B}_0$ , $C_0$ % as initial guess box and its center
Output: $C_i$ , $r_i$
$i \leftarrow 0$
while (1) do
    Create box $\mathsf{B_i}$ around $C_i$ and size of diam($\mathsf{B}_0$)
    $\mathsf{B_C} \leftarrow \mathsf{B_i}$
    $i \leftarrow i+1$
    while (diam($\mathsf{B_C}$) $> \varepsilon$) do
        $\mathsf{B}_{\mathsf{C}_1}, \mathsf{B}_{\mathsf{C}_2} \leftarrow$ Bisect($\mathsf{B_C}$) by the largest edge
        Calculate distance from $\mathsf{B}_{\mathsf{C}_1}$ and $\mathsf{B}_{\mathsf{C}_2}$ to $\mathscr{L}_B$, results $\mathbf{D}_{\mathbf{C_1}}$ and $\mathbf{D}_{\mathbf{C_2}}$
        if min(inf($\mathbf{D}_{\mathbf{C_1}}$)) $>$ min(inf($\mathbf{D}_{\mathbf{C_2}}$)) then
            $\mathsf{B_C} \leftarrow \mathsf{B}_{\mathsf{C}_1}$
            $\mathbf{D} \leftarrow \mathbf{D}_{\mathbf{C_1}}$
        else
            $\mathsf{B_C} \leftarrow \mathsf{B}_{\mathsf{C}_2}$
            $\mathbf{D} \leftarrow \mathbf{D}_{\mathbf{C_2}}$
        end if
    end while
    Return $C_i \leftarrow$ Center($\mathsf{B_C}$) %As MSFC center
    Return $r_i \leftarrow$ min($\mathbf{D}$) %As MSFC radius
    if $C_i$ is not close to the edges of $\mathsf{B}_{\mathsf{i}-1}$ then
        break while
    end if
end while
Return $C_i \equiv C_f$ % MSFC center
Return $r_i \equiv r_f$ % MSFC radius
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the last generated box. The obtained point C_f is the center of MWC and the radius can be computed by calculating the distance of C_f to the set of the WFW boundaries.

Figure 4 represents the evolution of the initial guess box, B_0, given by the user, to find the optimal center point of the MWC, $C_4 \equiv C_f$. Initial guess box B_0 is prescribed, but as the obtained center C_1 is close to the edges of B_0, consequently, another generated box B_1 is created around C_1. The procedure has found another candidate for being center point, C_2, but as it is still close to the edges of the generated box B_1, then it can be readily inferred that center point still tends to go further and, based on the latter logic, a new box is created about C_2, called B_2. Finally, the last obtained center point ($C_4 \equiv C_f$) is far enough form the edges of last generated box (B_3), therefore it is the final and optimal center point of MWC.

6. CONCLUSION

This paper investigated a kinetostatic problem of planar cable-driven parallel robots, namely the wrench feasible workspace. The problem was addressed by proposing two interval-based algorithm. The first algorithm aimed at finding an optimal design where the y-coordinate of the position of one actuator was subject to be optimized for a prescribed circle, upon considering wrench-feasible workspace. The second algorithm obtained the corresponding maximal inscribed circle of the wrench-feasible workspace. Obtaining such a region is a inherent asset in designing such a cable-driven parallel robot since leads to a safer and more conservative workspace in which the mechanism may exhibit better kinetostatic performance. Ongoing works include the extension of the proposed algorithms to the optimum dimensional synthesis for spatial 6-DOF cable-driven parallel robots.

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