WRENCH WORKSPACES OF REDUNDANTLY ACTUATED SPATIAL PARALLEL MANIPULATORS WITH MINIMUM ALLOWABLE WRENCH CAPABILITIES

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ABSTRACT

The wrench capability for a redundantly actuated parallel manipulator is defined as a measure of the maximum forces and moments that a mechanism can apply/sustain at the end-effector for a given pose. In this paper, minimum allowable wrench capability is introduced to describe a subset of the wrench capabilities which contain all forces and moments required for a given task. Limits on the forces and moments which must be generated are described by the surface of a particular shape. Two commonly employed shapes are the hyper-parallelepiped and hyper-ellipsoid. The usable workspace for this task is denoted as the wrench workspace and is a subset of the reachable workspace. Inside the wrench workspace, the forces and moments required for the task can always be generated. A numerical example is provided using the 3-RRRS redundantly actuated spatial parallel manipulator, and pure force, pure moment, and wrench workspaces are computed and compared. The algorithm presented here is shown to easily map to multiple processors for computation.

Keywords: wrench workspace; redundancy; zonotope.

ESPACES DES TORSEUR DES MANIPULATEURS PARALLÈLES AVEC REDONDANCE D'ACTIONNEMENT UTILISANT LES TORSEURS À CAPACITÉ MINIMALE

RÉSUMÉ

La capacité de forces et couples (torseurs) pour un manipulateur parallèle avec actionnement redondant peut être définie comme une mesure des forces et des moments maximaux permissibles que le mécanisme peut appliquer/soutenir à l'effecteur pour une pose de la nacelle donnée. Dans cet article, la capacité minimale des torseurs est introduite pour décrire un sous-ensemble de l'espace de travail qui représente, au minimum, le torseur minimum désiré. Ces torseurs minimaux peuvent être représentés par une forme géométrique quelconque, où l'utilisation d'une forme particulière dépend des applications souhaitées pour un manipulateur. Deux formes communément utilisées sont l'hyper-parallélépipède et l'hyper ellipsoïde. Une tâche donnée peut être prévue d'utiliser les forces et les moments décrits par la capacité du torseur minimal. L'espace utilisable pour cette tâche est l'espace de travail des torseurs qui est un sous-ensemble de l'espace de travail accessible. L'algorithme présenté dans cet article est facilement appliqué à plusieurs processeurs de calcul en parallèle. Un exemple numérique est fourni en utilisant le manipulateur parallèle 3-RRRS à actionnement redondant. L'espace de travail de force pure, de moment pur ainsi que l'espace de travail des torseurs sont calculés et comparés. L'algorithme présenté peut être facilement utilisé sur les processeurs de calcul en parallel.

Mots-clés : espace de travail ; redondance d'actionnement ; zonotope.

NOMENCLATURE

		Subscripts	
$\mathbf{F}$	Wrench vector		
$\boldsymbol{\tau}$	Active joints torques	HP	Hyper-parallelepiped
m	Number of active joints	HE	Hyper-ellipsoid
n	DOF of task		
$\mathcal{T}$	Task wrench set		
$\mathcal{A}$	Available wrench set		
$\mathcal{W}$	Actuator force workspace		
f_x, f_y, f_z	Principal force components		
m_x, m_y, m_z	Principal moment components		
$\mathbf{J}$	Jacobian matrix		
$r_{f_x}, r_{f_y}, r_{f_z}, r_{mx}, r_{my}, r_{mz}$	Semi-principal axes lengths		
$\mathbf{N}$	Normal vector matrix		
$\mathbf{V}$	Vertex matrix		
$\mathbf{D}$	Offset matrix		

1. INTRODUCTION

Parallel manipulators (PMs) are closed-loop mechanisms consisting of a moving platform connected to a fixed base by two or more serial chains (limbs). Compared with serial manipulators, PMs tend to have higher load-carrying capacity, rigidity, kinematic accuracy, and accelerations, making them useful for many applications (*e.g.*, [1, 2]). However, they suffer from small workspaces, complex forwards kinematics, low manoeuverability and highly singular workspaces [2]. Research currently focuses on the use of redundancy to overcome these drawbacks.

Redundancy occurs when the number of active joints, m, is greater than the dimensionality of the task, n. Spatial tasks require control over three dimensions of both forces and moments, termed a wrench, $\mathbf{F}$, and is defined as follows:

$$\mathbf{F} = [f_x, f_y, f_z, m_x, m_y, m_z]^T, \mathbf{F} \in \mathbf{R}^n \tag{1}$$

Redundancy in parallel manipulators can be divided into three main groups. Actuation redundancy [2] occurs when normally passive joints are replaced by active ones. The kinematic architecture of the manipulator does not change with this type of redundancy and the reachable workspace of the manipulator is unaffected. However, the wrench capabilities are affected [3] and forces of greater magnitudes can be generated. Branch redundancy (*e.g.*, cable actuated manipulators) also results in improved force capabilities. However, since additional active kinematic branches are added, this often results in reduced reachable workspace for manipulators with rigid links. Kinematic redundancy [2] adds to the mobility of the manipulator and results in an infinitude of possible solutions to the Inverse Displacement Problem (IDP). This type of redundancy occurs when extra active joints and links are added to a manipulator. Advantages can include larger reachable workspaces, avoidance of kinematic singularities, and dexterity improvement [2]. Merlet [4] also states the importance of redundancy in solving the forward kinematics, avoiding singular configurations, and improving obstacle avoidance, calibration, and force control.

Accomplishing a specific task requires the manipulator to be able to generate a given set of wrenches at a particular pose. Many papers have been published which use the terms force capability [5], force-moment capability [3, 6], and wrench capability [7–10] to loosely define the method of measuring the forces and/or moments that a mechanism can generate at a particular pose. In an attempt to generalize the terminology and definition of these capabilities, this paper uses the term “wrench capability” to define a measure of the maximum forces and moments that a mechanism can apply/sustain at the end-effector for a given pose. For the purpose of this paper, the concept of a “minimum allowable wrench capability” is also introduced. It describes a subset of the wrench capabilities which exhibit all forces and moments required for a specific

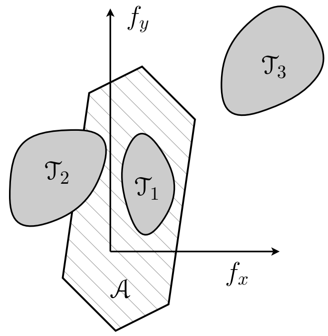


Fig. 1. Planar mechanism with associated $\mathcal{A}$ and three different $\mathcal{T}$ examples (adapted from [11]).

task. These are represented in the form of a geometric shape, where the shape's surface defines the limits of forces and moments that can be generated at the end-effector. Some of the most relevant shapes for parallel manipulators are points, polytopes (namely hyper-parallelepipeds), and hyper-ellipsoids, as described in [11]. In order for a pose to be able to generate the wrenches required for a task, it must be able to generate wrenches equal to or exceeding those defined by the minimum allowable wrench capabilities. Thus, if the minimum allowable wrench capabilities are not met or exceeded at at pose, that pose is not suitable for the given task.

As introduced in [11], the set of desired wrenches that must be generated for a given task at a particular pose is defined as the "Task Wrench Set", $\mathcal{T}$. Another set is used to describe all of the wrenches that a manipulator can generate in a given pose, and is defined as the "Available Wrench Set", $\mathcal{A}$. Set $\mathcal{A}$ depends primarily on the kinematic architecture and pose of the manipulator and the torque/force limits of each active joint. Similar sets have been used for cable-driven manipulators in [11–13]. The minimum allowable wrench capabilities at a given pose can be represented by $\mathcal{T}$. A particular pose can generate the minimum allowable wrench capabilities for a task if the following condition is met:

$$\mathcal{T} \subset \mathcal{A} \tag{2}$$

That is, if $\mathcal{T}$ is a subset of $\mathcal{A}$, then all wrenches on the surface of $\mathcal{T}$ or inside $\mathcal{T}$ can be generated by the manipulator. Figure 1 shows $\mathcal{A}$ and three different $\mathcal{T}$ for a planar mechanism with $n = 2$. Of the three $\mathcal{T}$, eq. (2) only holds true for $\mathcal{T}_1$. This is not the case if $\mathcal{T}$ is partially inside $\mathcal{A}$ (*e.g.*, $\mathcal{T}_2$) or completely outside of it (*e.g.*, $\mathcal{T}_3$).

When eq.(2) is valid, the wrenches described by $\mathcal{T}$ can be generated at the end-effector; however, they may not represent the maximum wrenches that can be generated. That is, the wrench capabilities may exceed the minimum allowable wrench capabilities. $\mathcal{T}$ can be then used as a standard means of ensuring a task can be accomplished at discrete poses throughout the reachable workspace. If the shape and size of $\mathcal{T}$ is fixed for a particular task, then various poses can be iterated through to determine a subset of the reachable workspace within which eq. (2) always holds true. The resulting workspace is known as the "wrench workspace" of the manipulator. Two other workspaces will also be referred to in this paper, namely the "pure force workspace" and the "pure moment workspace". These are special types of wrench workspaces, in which no moments or no forces are generated, respectively.

2. COMPUTING THE AVAILABLE WRENCH SET

Each active joint is capable of supplying a force/torque τ_i such that $\tau_{i\min} \leq \tau_i \leq \tau_{i\max}$. If $\boldsymbol{\tau}$ is a vector containing the actuator forces/torques of all active joints, where $\boldsymbol{\tau} \in \mathbb{R}^m$, then a linear mapping from the

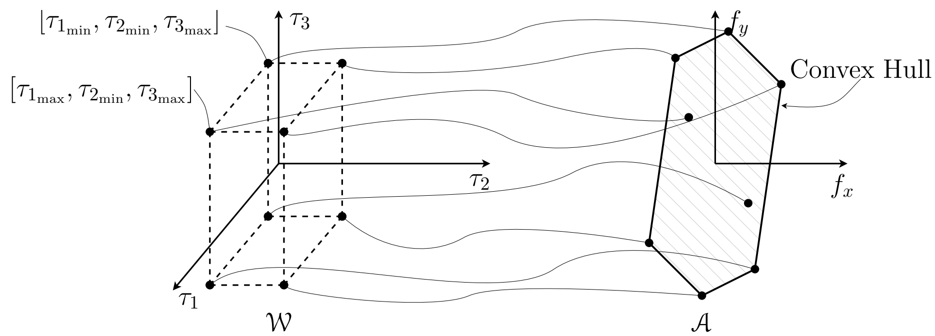


Fig. 2. Actuator Force Workspace (left) formed from the torque limits of three actuators (τ_1, τ_2, τ_3) mapped to form the Available Wrench Set (right) using the Jacobian matrix transformation [7].

joint space to the task space is accomplished using:

$$\mathbf{F} = (\mathbf{J}_q^{-1}\mathbf{J}_x)^T \boldsymbol{\tau} \tag{3}$$

where, $\mathbf{J} = \mathbf{J}_q^{-1}\mathbf{J}_x$ is the $m \times n$ screw-based Jacobian matrix, as defined by Tsai in [14].

In non-redundant manipulators $n = m$, thus $\mathbf{J}$ is square and therefore eq. (3) represents a unique mapping from $\boldsymbol{\tau}$ to $\mathbf{F}$. This is not the case for redundantly actuated manipulators, where $n < m$, as eq. (3) is no longer unique. Instead, there can be multiple combinations of $\boldsymbol{\tau}$ which can produce exactly the same $\mathbf{F}$.

As shown by Carretero and Gosselin in [7], all possible joint torque combinations that can be generated by the manipulator can be defined as the "Actuator Force Workspace", denoted $\mathcal{W}$. This is represented geometrically as a hyper-parallelepiped, as shown in Figure 2, where each facet represents a torque limit imposed by $\tau_{i\text{min}}$ or $\tau_{i\text{max}}$ on a particular actuator. Each vertex of the hyper-parallelepiped represents all m actuators working at their respective maximum or minimum torque limits. Since $\mathcal{W}$ is a m-dimensional convex polytope – in fact a zonotope (centrosymmetrical convex polytope) – the linear mapping performed by $\mathbf{J}$ yields a n-dimensional zonotope. That is, the linear mapping of a convex space also produces a convex space, and therefore only the vertices of $\mathcal{W}$ must be mapped, as shown in Figure 2. A n-dimensional convex hull routine – the smallest convex set that contains those points – can then be used to obtain $\mathcal{A}$ from the set of mapped points. This mapping is uni-directional, as not all vertices of $\mathcal{W}$ are vertices of $\mathcal{A}$, thus the vertices of $\mathcal{A}$ cannot be mapped backwards to obtain $\mathcal{W}$. This work currently uses the *qhull* [15] algorithm for computing the convex hull. Bouchard *et al.* [2] describe a more computationally efficient and non-iterative convex hull routine, especially for spatial manipulators, which they refer to as the hyperplane shifting method. Gouttefarde and Krut [16] provide a proof which leads directly to an improved version of this method.

3. MINIMUM WRENCH CAPABILITIES

Set $\mathcal{A}$ gives all of the possible wrenches that can be produced by the manipulator, where the surface of the zonotope bounds the maximum wrenches that can be generated. This paper considers two techniques for analysis of the minimum allowable wrench capabilities where $\mathcal{T}$ is defined as:

– A n-dimensional axis-aligned hyper-ellipsoid (HE), denoted by ($\mathcal{T}_{HE}$) and given for the spatial case by:

$$\frac{f_x^2}{r_{f_x}^2} + \frac{f_y^2}{r_{f_y}^2} + \frac{f_z^2}{r_{f_z}^2} + \frac{m_x^2}{r_{m_x}^2} + \frac{m_y^2}{r_{m_y}^2} + \frac{m_z^2}{r_{m_z}^2} = 1 \tag{4}$$

where, r_* ($*$ represents all applicable subscripts) represent the length of the axis-aligned semi-principal axes of $\mathcal{T}_{HE}$ for forces and moments denoted by subscripts f and m respectively.

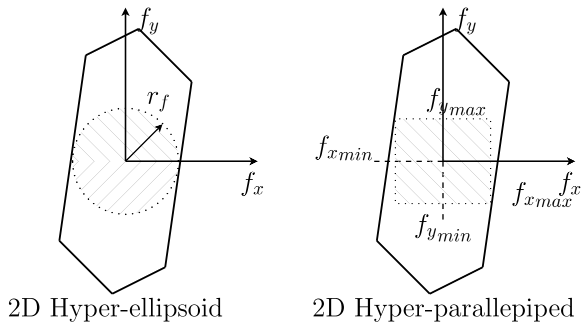


Fig. 3. Minimum allowable wrench capabilities with the Task Wrench Set defined as a hyper-ellipsoid (left) and hyper-parallelepiped (right) for a 2-dimensional case (*i.e.*, $n = 2$).

The independent lengths for the force and moment semi-principal axes account for the mixed units in **F**. This method, and projections of this method for planar cases, with $r_{f_x} = r_{f_y} = r_{f_z}$ and $r_{mx} = r_{my} = r_{mz}$, has been used in many papers [5, 7, 11], although some do not report on the geometrical significance. The popularity of this technique is because it provides knowledge of the maximum magnitude of forces and moments that the manipulator can sustain irrespective of the direction, making it useful for design purposes.

– A n-dimensional axis-aligned hyper-parallelepiped (HP), denoted ($\mathcal{T}_{HP}$), defined by the boundaries $B_{HP\min}$ and $B_{HP\max}$ for the spatial case as:

$$\begin{aligned} B_{HP\min} &= [f_{x\min}, f_{y_{\min}}, f_{z\min}, m_{x\min}, m_{y_{\min}}, m_{z\min}]^T \\ B_{HP\max} &= [f_{x\max}, f_{y_{\max}}, f_{z\max}, m_{x\max}, m_{y_{\max}}, m_{z\max}]^T \end{aligned} \tag{5}$$

Similar to the Actuator Force Workspace, the facets of $\mathcal{T}_{HP}$ corresponds to the force and moment limits imposed by eq. (5), while the vertices correspond to the 2^n possible combinations of boundary limits on their respective principal axes. That is, one boundary limit (either min or max) is selected for each of the n principal axes. Each combination results in one vertex on $\mathcal{T}_{HP}$. This technique becomes useful when a task requires particular forces or moments in certain directions, which may not necessarily be symmetric about the origin.

For clarity, consider the case of a planar manipulator where only the forces f_x and f_y and moment m_z can be generated. That is, equations (4) and (5) are projected onto a plane where $f_z = m_x = m_y = 0$. If we consider only the forces for this manipulator, (*i.e.*, $m_z = 0$), $\mathcal{T}_{HE}$ becomes an origin-centred circle on the f_x–f_y plane with radius $r_f = r_{f_*}$, as follows:

$$f_x^2 + f_y^2 = r_f^2 \tag{6}$$

Since any point located on the surface or inside this circle can be generated, it can be concluded that a force of magnitude r_f can be applied in any direction on the f_x–f_y plane, as shown in Figure 3. If we again consider the planar manipulator case and only look at the forces, $\mathcal{T}_{HP}$ simplifies to a rectangle bounded by:

$$\begin{aligned} B_{HP\min} &= [f_{x\min}, f_{y_{\min}}]^T \\ B_{HP\max} &= [f_{x\max}, f_{y_{\max}}]^T \end{aligned} \tag{7}$$

Any force can be generated as long as f_x and f_y remain within the bounds defined by $B_{HP\min}$ and $B_{HP\max}$.

The values of r_{f_*} and r_{m*}, as well as $B_{HP\min}$ and $B_{HP\max}$, are fixed for the computation of the minimum allowable wrench capabilities. The bounding parameters of these two techniques may be further increased until the point where $\mathcal{T}$ becomes inscribed. Any additional increase in the wrench magnitude invalidates eq. (2).

Since zonotopes can be represented as either a vertex or hyperplane representation ($\mathcal{V}$ and $\mathcal{H}$ respectively), validation of eq. (2) can be simplified using the $\mathcal{H}$-representation for $\mathcal{A}$ and the $\mathcal{V}$-representation for $\mathcal{T}$ (with a slight modification for $\mathcal{T}_{HE}$). This validation can be performed by verifying the following equation:

$$\mathbf{NV} \leq \mathbf{D} \tag{8}$$

where, $\mathbf{N}$ is a matrix whose rows are the unit normal vectors of the supporting hyperplanes, $\mathbf{V}$ is a matrix where each column is a vertex of $\mathcal{T}_{HP}$ or point on the surface of $\mathcal{T}_{HE}$ whose normals are identical to those of the supporting hyperplanes, and $\mathbf{D}$ is a matrix whose rows are the projected offsets of the corresponding hyperplanes along their respective normal vector. The reader should refer to Bouchard *et al.* [11] for additional insight into the formulation of eq. (8).

4. WRENCH WORKSPACE

Performing a task requires a manipulator to be able to generate certain forces and moments, as well as to be able to continuously apply/sustain them throughout a trajectory. This leads to the idea of a wrench workspace, which describes the usable space within the reachable workspace of a manipulator where a particular task can be performed. For simplicity, a constant orientation of the end-effector can be used to determine the wrench workspace.

The search for the wrench workspace boundary can be accomplished by using a cylindrical grid to discretise the reachable workspace into a set of search vectors, similar to that used by Garg *et al.* [6]. To do this, the reachable workspace is sliced into planes of constant elevation, where each of these planes is then sliced into equal segments about its relative origin. Each segment is given a search vector which is used to determine the corresponding boundary of the wrench workspace along that vector. This can be accomplished by performing a line-search outwards from the origin along each search vector until the edge of the wrench workspace is accurately detected. It is important to note that this search technique may not provide accurate results if the reachable workspace contains voids, if the search vector encounters a singular configuration, or if the wrench capabilities do not decrease radially outwards. Thus, unless a more robust search technique is used, proper knowledge of the reachable workspace is required to obtain accurate results.

Each search vector performs a completely independent calculation and no sharing of information is required between the search vectors for determining the workspace boundaries. This allows the computations to easily be mapped to multiple processors for performing parallel computations, and furthermore can be classified as an embarrassingly parallel algorithm.

5. NUMERICAL EXAMPLES

This section gives the results for wrench workspaces of the 3-<u>RRR</u>S redundantly actuated spatial parallel manipulator, as defined by Garg *et al.* [6], using the $\mathcal{T}_{HE}$ and $\mathcal{T}_{HP}$ techniques. For the purpose of these examples, the topology and geometry of the manipulator is identical to that used in [6], and is shown in Figure 4. That is, $g_* = h_* = r_p = 1$m, $r_b = 2$m, $\alpha = 120^\circ$, and $\beta = 240^\circ$. Each joint is given identical torque limits of ± 1Nm (*i.e.*, $-\tau_{ij_{\min}} = \tau_{ij_{\max}} = 1$Nm, for $i = 1,2,3$ and $j = 1,2,3$). This manipulator does not contain any voids inside its workspace and its singular configurations are accounted for during runtime.

Examples of various wrench workspaces are obtained by defining the boundaries of $\mathcal{T}$ as ± 1.4N for forces and ± 1.0Nm for moments. For $\mathcal{T}_{HE}$, this is represented by setting $r_{f_*} = 1.4$N and $r_{m*} = 1.0$Nm in eq. (4), while for $\mathcal{T}_{HP}$, the force and moment components of $B_{HP\min}$ and $B_{HP\max}$ are set to their respective values.

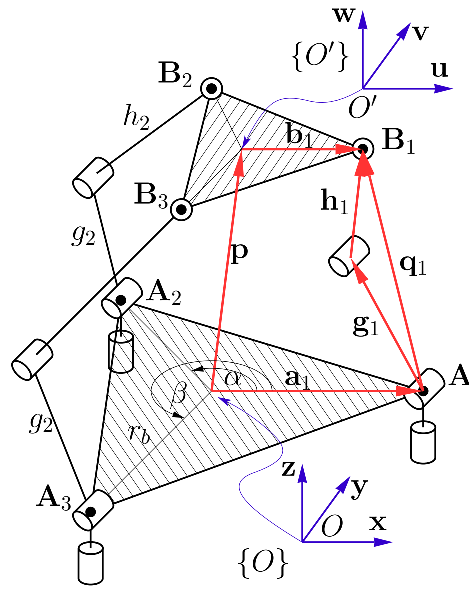


Fig. 4. 3-RRRS redundantly actuated parallel manipulator architecture, as denoted in [6].

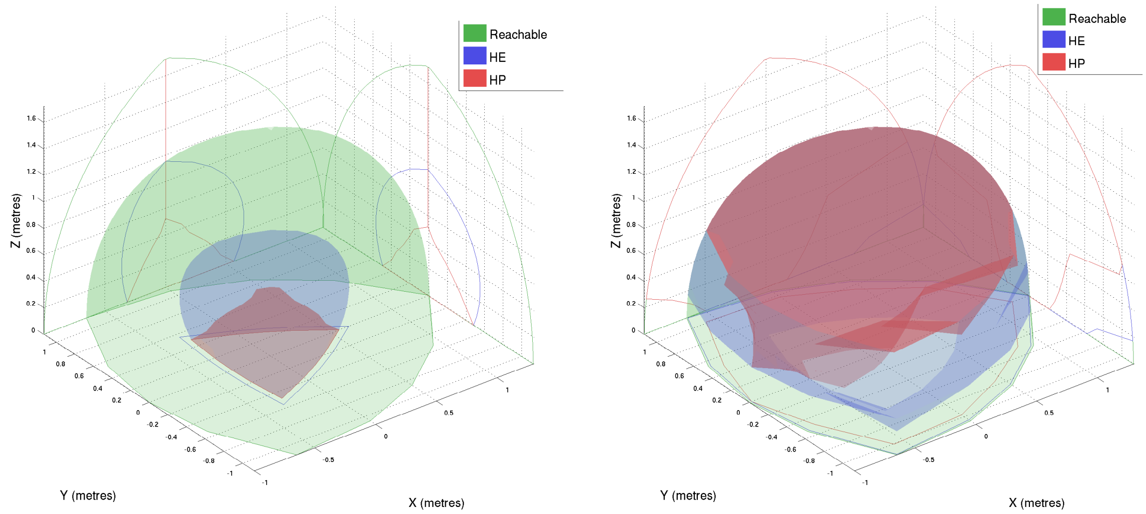
Results are also shown for both pure force and pure moment workspaces. For a pure force workspace, the boundaries of the moment components are set to zero, and *vice-versa* for a pure moment workspace.

Results for the $\mathcal{T}_{HE}$ and $\mathcal{T}_{HP}$ pure force and pure moment workspaces, compared against the reachable workspace, are shown in Figure 5. As expected, the $\mathcal{T}_{HP}$ workspaces are smaller than those using $\mathcal{T}_{HE}$. Considering the actual geometric shapes of $\mathcal{T}_{HE}$ and $\mathcal{T}_{HP}$ for the considered case, and noting that the boundaries are identical, it can be verified that $\mathcal{T}_{HE} \subset \mathcal{T}_{HP}$. Thus, when the workspaces resulting from the two techniques are considered, the $\mathcal{T}_{HP}$ workspace is a subset of the $\mathcal{T}_{HE}$ workspace. An important distinction between the $\mathcal{T}_{HE}$ and $\mathcal{T}_{HP}$ workspaces, are their symmetries. That is, while the $\mathcal{T}_{HE}$ workspace exhibits the same symmetric properties as the 3-RRRS manipulator itself, the $\mathcal{T}_{HP}$ workspace does not exhibit the same symmetric properties. This is the case as $\mathcal{T}_{HP}$ is defined relative to the fixed base frame, so any reorientation of $\mathcal{T}_{HP}$ describes completely different minimum allowable wrench capabilities. Actually, the same issue can be stated about any convex geometric shape other than the hyper-ellipsoid used in these examples, because any rotation of an origin-centred sphere yields the same shape.

Using only $\mathcal{T}_{HE}$, Figure 6 shows the results for a pure moment, pure force, and wrench workspace compared with the reachable workspace. The pure force and pure moment workspaces are nearly identical to those presented by Garg *et al.* [6], which were obtained using the same criteria. Application of the wrench workspace to PM design is an important achievement, as the wrench workspace is a subset of both the pure force and pure moment workspaces, and represents the usable space within which desired forces and moments can always be generated. Considering the case of a lifting application, a force is required for translation of an object, while a moment is required for any object eccentricity and dynamic effect. It is very rare that either will be used alone, thus the importance of the wrench workspace should not be understated. Maximizing the size of the wrench workspace will allow for the design of wrench-optimised PMs.

6. CONCLUSIONS

The minimum allowable wrench capabilities for a redundant parallel manipulator are shown to be important in determining the pure force, pure moment, and wrench workspace. Defining the Task Wrench Set as either a hyper-ellipsoid or hyper-parallelepiped yields important information about the capabilities of a ma-



(a)

(b)

Fig. 5. Comparison of pure force (a) and pure moment (b) workspaces for the $\mathcal{T}_{HP}$ and $\mathcal{T}_{HE}$ wrench capability techniques.

nipulator in generating forces and moments for a particular task. The hyper-ellipsoid workspaces are shown to exhibit the same symmetric properties as the manipulator, while the hyper-parallelepiped workspaces are orientation-dependant. The algorithm presented can easily be mapped to multiple processors for computation, and further improvements can be gained using the hyper-plane shifting method. Importance of the wrench workspace as a design criteria for future parallel manipulators may allow for wrench-optimised designs. This would allow manipulators to be optimised for a particular task, where the majority of the workspace becomes usable for that task. Future work will consider improvements to the wrench workspace boundary detection routine, accounting for singular configurations within the reachable workspace, and the development of a parallel optimisation algorithm for designing wrench-optimised parallel manipulators. The concept of an interval-based algorithm will also be considered as a possible improvement.

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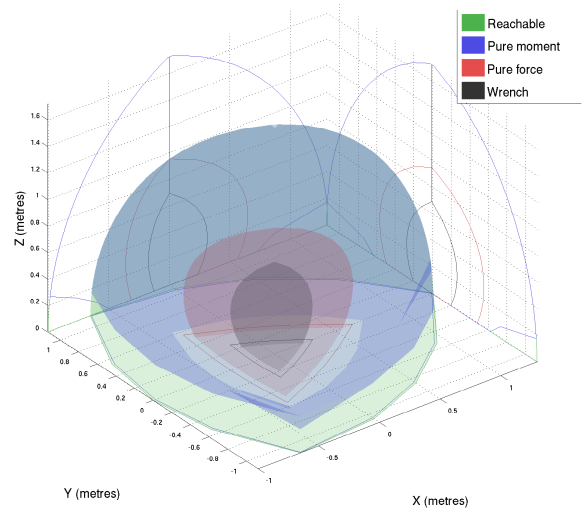

Fig. 6. $\mathcal{T}_{HE}$ wrench workspaces showing pure force, pure moment, wrench, and reachable workspaces.

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