INVESTIGATION OF THE KINEMATIC SENSITIVITY OF 3-R$\underline{P}$R PARALLEL MECHANISM

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ABSTRACT

In the process of designing a robot, it is of paramount importance to define a kinetostatic performance criterion, where various performance indices are proposed for this purpose. These indices should be valid and correspond to the practical environment. The main goal of this paper is to provide a comprehensive investigation for the criteria proposed in the literature, based on the physical characteristics of the problem. The concept of this paper can be well extended to general parallel mechanisms. Among the suggested performance indices, dexterity, manipulability, condition number, and also the recently proposed, maximum point-displacement and rotational kinematic sensitivity are the most popular ones and are subject to be analyzed. In this paper, first according to the kinematic uncertainties, some performance indices are reviewed. Then, according to these indices, their ability to predict the errors for a 3-R$\underline{P}$R parallel mechanism is considered. That is, a finite error is assumed in the joint space and the resulted error in the Cartesian space of the moving platform is compared with the values returned by these indices. Finally, it has been concluded that the maximum point-displacement and rotational kinematic sensitivity return more valid interpretation.

Keywords: 3-R$\underline{P}$R planar parallel mechanisms; point-displacement kinematic sensitivity; rotational kinematic sensitivity; kinetostatic performance indices.

LA SENSIBILITÉ CINÉMATIQUE D'UN MÉCANISME PARALLÈLE 3-R$\underline{P}$R

RÉSUMÉ

Dans le processus de conception d'un robot, il est d'une importance primordiale de définir un critère de performance cinétostatique, où diverses performances indices sont proposées à cet effet. Ces indices doivent être valides et correspondre à l'environnement pratique. L'objectif principal de cet article est de fournir une enquête approfondie pour les critères proposés dans la littérature, en fonction des caractéristiques physiques du problème. Parmi les indices de performance proposées, la dextérité, maniabilité, le conditionnement, et aussi l'indice récemment proposé, la sensibilité cinématique de translation et de rotation sont les les plus populaires et font l'objet d'analyser de cette étude. Dans cet article, d'abord selon les incertitudes cinématiques, certains indices de performance sont examinés. Puis, selon ces indices, leur capacité à prédire l'erreur pour un mécanisme parallèle 3-R$\underline{P}$R est pris en compte. Autrement dit, une erreur finie est supposée dans l'espace articulaire et l'erreur entraînée dans l'espace cartésien de la plate-forme mobile est comparée avec les valeurs calculés par ces indices. Enfin, il a été conclu que la sensibilité cinématique de translation et rotation une interprétation plus valide.

Mots-clés : mécanismes parallèles planaires à 3-ddl ; la sensibilité cinématique de translation ; la sensibilité cinématique de rotation ; indices de performances cinématiques.

1. INTRODUCTION

Parallel Mechanisms (PMs), due to some of their remarkable kinetostatic properties, nowadays are developing rapidly and becoming the-state-of-the art in wide range of industrial and academical applications such as motion simulators [1], Nano mechanisms [2, 3] and many other devices and parallel kinematic machines [4]. PMs are well known for their ability to perform trajectory that demand high precision in terms of their kinematic and dynamic properties. The potency of PMs is more apparent when a relatively high payload with respect to the size of the robot should be carried by the moving platform. A flight simulator can be regarded as a solid instance for this concept, where using a Gough-Stewart platform is a dominant concept in this field. However, recently, their kinematic precision has been questioned and is becoming the subject of many critical reviews reported in literature [5, 6]. This property can be affected by various environmental or structural circumstances, such as design errors, flexibility of the links, thermal expansion and etc. [7]. It should be noted that the study conducted in this paper, can be well extended to all general PMs, but here, more emphasis is placed on a 3-R$\underline{\text{P}}$R Planar Parallel Mechanism (PPM).

Among several indices suggested for the purpose of comparison and design of PMs, there are some cases which do not address the effects of the input error as a single criterion [8], i.e., they measure the performance of the PMs, by blending the input errors and the design errors. However, recently, a kinetostatic index, referred to as kinematic sensitivity, has been proposed, which measures the effects of input errors in the Cartesian space of PMs. In this paper, kinematic sensitivity is subject to be investigated and examined for coincidence with the actual calculated error, in this paper. As stated in [7], up to now, a consensus among the robotics community has not been drawn on the latter issue. The main challenge, which is a major deterrent in filling the latter gap, consists in establishing an appropriate mathematical definition of uncertainties, i.e., a proper selection of the norms involved in the problem, which are present in practice. In fact, none of the proposed indices meet the required physical applicability, as the unit-inconsistent quantities cannot be merged and normalized together.

In fact, the robustness of a PM against the kinematic uncertainties depends on a set of geometric properties associated to the mechanism [5]. As a result, different performance indices have been developed in order to optimize and compare PMs in order to propose the most promising design for prescribed tasks and motion patterns [9, 10].

Among various performance indices proposed for robotic mechanical systems, PMs, among others, the most popular ones are manipulability [11], dexterity [12] and condition number [13]. By the way, a general PM performs both translations and rotations and consequently, its Jacobian matrix compromises dimensionally non-homogeneous components. Therefore, as the aforementioned indices normalize the unit-inconsistent variables, changing the scale of the variables may change the result significantly and thus they do not return physically applicable values about the performance of PMs [5, 14–17].

Furthermore, from the above, one can conclude that some well-known indices, such as condition number, may not work properly, even for the cases where the PM performs pure translations or pure rotations [7]. In addition, some indices suffer from some other degenerations. For example, the manipulability index is invariant to the size of the moving platform and therefore, does not distinguish the difference between a large and a small moving platform, which is a serious limitation of this index [5, 18].

There are many perspectives promoted to overcome the latter insufficiencies and stimulated the interest of many researchers during the last decade. For instance, a weighted version of the manipulability index is suggested in [14]. Moreover, on the same regard, some attempts are made to circumvent the above problem by normalizing the Jacobian matrix, which is investigated in [15, 18–20] and more precisely the so-called characteristic length, proposed in [21]. By the way, all these methods are unable to result in physically applicable values, as the fact of unit-inconsistency of the components of the Jacobian matrices is not disputable.

The study carried out in [16] investigates the kinematic sensitivity index by analyzing two different types

of norms, namely 2-norm and ∞-norm. To this end, kinematic sensitivity is computed in three points of the workspace of a given Gough-Stewart platform, using various combination of the norms, and finally it has been concluded that in practice, the results returned by this index does not correspond to the calculated error. In order to circumvent the latter deficiency in the process of optimization, a maneuver has been proposed in [5] which considers two separate indices for translational and rotational Degree-of-Freedoms (DOFs). These two indices are called point-displacement and rotational kinematic sensitivity, respectively, which provide a reliable interpretation about the performance of robotic mechanical systems, PMs among others. The main contribution of this paper is providing an investigation upon putting into contrast the differences and credibility of these two indices with the others. The analytical and geometrical procedures for computing these indices are extensively surveyed in [22] and, as the fundamental contribution of this paper, the validity and applicability of them to the practical environments is investigated.
The remainder of this paper is organized as follows. First, we touch upon some preliminary concepts of kinematic analysis, such as Forward Kinematic Problem (FKP), Inverse Kinematic Problem (IKP) and the Jacobian matrix of the mechanism under study, i.e., the 3-R$\underline{\text{P}}$R PM. The paper pursues the study by investigating some performance indices, and also among them the maximum kinematic sensitivity, considering the relation between the joint rate space and the Cartesian velocity space. Then, two different criteria are introduced, upon which, the reliability and coincidence of performance indices to the calculated error of the workspace of the PM can be examined. Subsequently, the paper continues with applying the involved performance indices to the 3-R$\underline{\text{P}}$R PPM and compares the credibility of the indices, according to the degree of coincidence of the outputs of the latter indices to the calculated error of the PPM. Finally, the paper concludes by specifying some enlighten deduction about the degree of authenticity of the concerned performance criteria.

2. KINEMATIC MODELING OF THE 3-R$\underline{\text{P}}$R PARALLEL MECHANISM

As aforementioned, most performance indices proposed in the domain of optimization and comparison of PMs are defined on the basis of input-output velocity relation which is tantamount to the concept of the Jacobian matrix. Thus, calculation of these indices necessitates a thorough review of the first-order kinematic, and then, IKP and FKP of PMs, which is surveyed comprehensively in many studies, such as [17] and [23]. Thus their results are to be broadly reviewed in what follows.

2.1. Kinematic Modeling, First Order Relations

Figure 1 depicts schematically a 3-DOF 3-R$\underline{\text{P}}$R PM . The fixed base and moving platform are defined by $\Delta A_1A_2A_3$ and $\Delta B_1B_2B_3$, respectively. The R passive joints are connected to A_i and B_i, $i = 1,2,3$, where a prismatic actuator, with adjustable leg length as ρ_i, connect them. The unit vectors along the prismatic actuators are defined by $\mathbf{n}_i = \begin{bmatrix} n_{i_x} & n_{i_y} & 0 \end{bmatrix}^T$, whence $\boldsymbol{\rho}_i = \rho_i \mathbf{n}_i$. The position of B_i in the fixed and moving frame is denoted by $\mathbf{b}_i$ and $\mathbf{b}'_i$, respectively. It should be noted that, here and throughout this paper, $(\cdot)'$ stands for a representation of its vector argument in the moving frame. The rotation matrix performing the transformations from the fixed frame into the moving frame can be formulated as follows:

$$\mathbf{R} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{1}$$

where ϕ is the angle of the rotation of the moving frame with respect to the fixed frame. Considering $\mathbf{a}_i$ as the position of the point A_i in the fixed frame, with respect to the geometry of the robot, leads to:

$$\boldsymbol{\rho}_i = \mathbf{p} + \mathbf{R}\mathbf{b}'_i - \mathbf{a}_i, \tag{2}$$

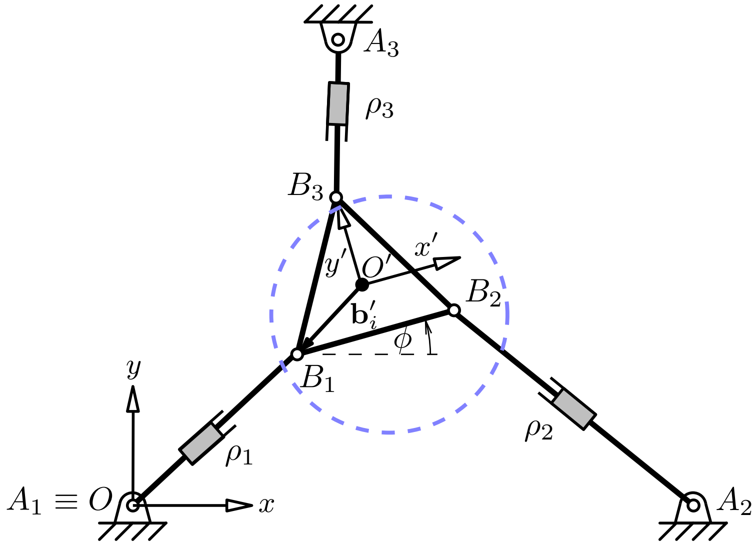

Fig. 1. The 3-R$\underline{\text{P}}$R planar parallel mechanism [24]. The circle indicates the points in which the error is sought to be calculated.

where $\mathbf{p}$ is the position vector of the center of the moving platform (point O') with respect to O, i.e., the origin of the fixed frame.

2.2. The IKP

According to Eq. (2), parameters $\mathbf{a}_i$, $\mathbf{b}'_i$, $\mathbf{p}$ and also the the angle ϕ are already known for calculating the IKP, thus one has the following for the IKP:

$$\rho_i = \|\boldsymbol{\rho}_i\|_2 = \sqrt{\left(\mathbf{p}+\mathbf{R}\mathbf{b}'_i-\mathbf{a}_i\right)^T\left(\mathbf{p}+\mathbf{R}\mathbf{b}'_i-\mathbf{a}_i\right)}, \tag{3}$$

where $\boldsymbol{\rho} = \begin{bmatrix}\rho_1 & \rho_2 & \rho_3\end{bmatrix}^T$ is the input vector, i.e., its components are the leg length of the prismatic actuators.

2.3. The FKP

In what concerns the FKP, parameters $\mathbf{a}_i$, $\mathbf{b}'_i$ and ρ_i are known and the problem pertains at finding $\mathbf{p}$ and ϕ, i.e., the pose (position and orientation) of the moving platform [24, 25], which construct the output vector $\mathbf{x} = \begin{bmatrix}x & y & \phi\end{bmatrix}^T$. Usually, finding a closed-form solution for the FKP of PMs is a complicated task which may initiate both mechanical and mathematical challenges. To this end, some numerical approaches are suggested, such as Newton-Raphson method and other complex mathematical maneuvers such as exploring the problem in seven-dimensional space, using the so-called Study's parameters [26]. The FKP of 3-R$\underline{\text{P}}$R PM is investigated in detail in [25, 26] and the results revealed that it admits up to six solutions, including complex and real ones.

In the study carried out through this paper, a numerical approach, following the guidelines of the trust-region-reflective method [27], is implemented to find the solutions to the FKP of the 3-R$\underline{\text{P}}$R since only one solution is sought to be obtained from the six possible one. To this end, Eq. (3), for $i = 1, \cdots, 3$, leads to a system of three equations and three unknowns x, y and ϕ, which should be solved upon the above mentioned numerical method, by having the input variables, i.e., ρ_1, ρ_2 and ρ_3.

2.4. Input-Output Velocity Relations, Jacobian Matrix

Jacobian matrix, arisen from the first order-kinematic relation, represents the mapping between input and output velocity (infinitesimal variation) vectors, namely $\dot{\boldsymbol{\rho}} = \begin{bmatrix}\dot{\rho}_1 & \dot{\rho}_2 & \dot{\rho}_3\end{bmatrix}^T$ and $\mathbf{t} = \begin{bmatrix}\dot{x} & \dot{y} & \dot{\phi}\end{bmatrix}^T$, respectively. Various approaches are proposed in order to obtain the Jacobian matrix of PM which is beyond the scope of this paper and is thoroughly elaborated in [24, 25]. However, for a general PM, the first-order kinematic relation can be formulated as follows:

$$\dot{\boldsymbol{\rho}} = \mathbf{K}\mathbf{t}, \tag{4}$$

where $\mathbf{K}$ is called the inverse Jacobian matrix. The above, when applied to the case of the 3-R$\underline{\text{P}}$R PM, results in:

$$\mathbf{K} = \begin{bmatrix} n_{1_x} & n_{1_y} & \left(\mathbf{R}\mathbf{b}'_1 \times \mathbf{n}_1\right).\mathbf{k} \\ n_{2_x} & n_{2_y} & \left(\mathbf{R}\mathbf{b}'_2 \times \mathbf{n}_2\right).\mathbf{k} \\ n_{3_x} & n_{3_x} & \left(\mathbf{R}\mathbf{b}'_3 \times \mathbf{n}_3\right).\mathbf{k} \end{bmatrix}, \tag{5}$$

where $\mathbf{k}$ is the unit vector along the z-axis [23].

3. KINEMATIC PERFORMANCE INDICES

The main aim of performance indices is to propose a design, upon which, a well-conditioned workspace, in terms of kinematic properties, can be obtained. Jacobian matrix can be used to determine a linear mapping between the errors in the joint space and the Cartesian space of the moving platform. This point has a great significance in calculating the performance indices, which is substantial in the field of optimal design of PMs. In what follows, some of these kinetostatic performance indices are to be reviewed.

As the first case, the manipulability performance index, μ, is considered, which can be expressed mathematically as follows:

$$\mathbf{t} = \mathbf{K}\dot{\boldsymbol{\rho}}, \qquad \mu = 1/\sqrt{\det\left(\mathbf{J}\mathbf{J}^T\right)}, \tag{6}$$

where $\mathbf{J}$ is the forward Jacobian matrix [11]. The value returned by this index is proportional to the volume of the manipulability ellipsoid.

Moreover, another popular index, the condition number, κ, is defined as:

$$\kappa = \|\mathbf{K}\|\|\mathbf{K}^{-1}\|, \tag{7}$$

which indicates the ratio of the error amplification from the joint space to the Cartesian space of the moving platform [13]. In fact, by considering the 2-norm, the value returned by this index is proportional to the ratio of the large diameter to the small one of the manipulability ellipsoid.

Having in mind the fact that the Jacobian matrix for a general PM is dimensionally non-homogeneous, the two performance indices defined above, may lead to erroneous interpretations about the performance of PMs, since changing the scale of components and variables in Jacobian matrix can change the final result significantly.

In order to overcome the latter problem, two separate indices for the translation and rotation parts of the Jacobian matrix have been recently proposed in [5], referred to as point-displacement and rotational kinematic sensitivity, respectively. These two indices can be physically interpreted as upper-bound limits for translational and rotational errors, respectively, which can formulated as follows:

$$\sigma_{p_{c,f}} = \max_{\|\dot{\boldsymbol{\theta}}\|_c=1} \|\mathbf{p}\|_f, \qquad \sigma_{r_{c,f}} = \max_{\|\dot{\boldsymbol{\theta}}\|_c=1} \|\phi\|_f, \tag{8}$$

where $\sigma_{p_{c,f}}$ and $\sigma_{r_{c,f}}$ are the point-displacement and rotational kinematic sensitivity, respectively. In addition, c and f are the norm of the joint rate vector and the norm of the moving platform pose vector, respectively.

As pointed out previously, these two indices consider the translational and rotational errors, separately and therefore are unit-consistent. Two types of norms which are most relevantly used for c and f, are 2 and ∞. Therefore, four different combinations of the latter norms are possible for calculating the kinematic sensitivity, where all of them are mentioned and compared in [22] and finally it has been concluded that $c=\infty$ and $f=2$ bears the most meaningful and physically applicable conception about the performance of the PMs.

The most of well-known kinematic performance indices are classified as posture-dependent indices in [18, 28]. By the way, comparison and optimization of PMs which are based on this kind of indices may lead to erroneous conclusions. Thus, a global index is required, which returns a value describing the kinetostatic efficiency of a PM, with regard to the whole feasible singularity-free workspace. In the case of kinematic sensitivity, an index, called global kinematic sensitivity is suggested in [29, 30], where it follows that:

$$\zeta_l = \frac{\int_W l dW}{\int_W dW} \Longrightarrow \qquad \zeta_{\sigma_{p_{\infty,2}}} = \frac{\int_W \sigma_{p_{\infty,2}} dW}{\int_W dW}, \qquad \zeta_{\sigma_{r_{\infty,2}}} = \frac{\int_W \sigma_{r_{\infty,2}} dW}{\int_W dW}. \tag{9}$$

The above formulation is used for the purpose of this paper. It should be noted that in the above dW is a differential element of the workspace.

4. CREDIBILITY ANALYSIS AND COMPARISON OF KINEMATIC SENSITIVITY INDICES

The kinematic performance indices are presumed to return values which correspond to the practical reality. As a consequence, PMs designed and optimized in virtue of these indices as objective criteria, leads to possibly one of the best decisions about the architecture properties.

It is worth noticing that the performance index can be a proportional representative of the actual error occurred in the workspace of the PM under study. Assume I_1 as a valid index, representing the PM workspace error, as depicted in Fig. 2. Considering the arbitrary constant α_1, and then assuming $I_2 = \alpha_1 I_1$ as another index, one has:

$$\zeta_{I_2} = \alpha_1 \zeta_{I_1}, \tag{10}$$

where ζ_{I_1} and ζ_{I_2} are the global indices for I_1 and I_2, respectively, defined by the manner illustrated in Eq. (9). Thus, comparing two PMs on the basis of the new index I_2, leads to the same inference.

Also the performance index can be either higher or lower than the actual index, with a constant offset. More precisely, consider another arbitrary constant, named α_2, and assume $I_3 = I_1 + \alpha_2$ as another index, as represented in Fig. 2. According to Eq. (9), one has:

$$\zeta_{I_3} = \zeta_{I_1} + \alpha_2, \tag{11}$$

where ζ_{I_3} is the global index for I_3, defined by the manner illustrated in Eq. (9). Thus, comparing two PMs on the basis of the new index I_3, also results in the same overall interpretation.

4.1. Performance Criteria

In what follows, two kinematic criteria are investigated, which are applicable to all PMs and no particular PM is considered from the outset.

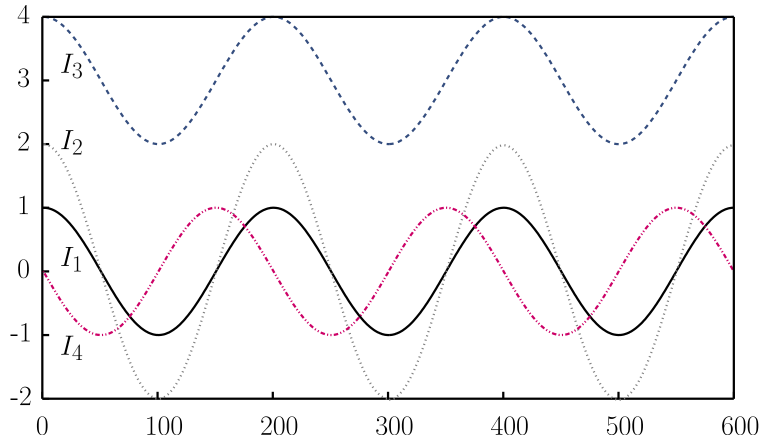

Fig. 2. The valid index, I_1 (-), and the imaginary indices, I_2 (-.-), I_3 (–) and I_4 (.).

Point⇒ Error criterion⇓	P_1	P_2	P_3
Real error	0.2	0.3	0.8
Index I_1	5	4	9
Index I_2	200	310	2000

Table 1. The error of a PM in three particular points, P_1, P_2 and P_3, considering the calculated errors and two indices I_1 and I_2.

4.1.1. The First Criterion

A suitable index should save the order of the extent of the error of the points in which it is measured. More precisely, it should return a higher value, when the error is increased in a point and *vice versa*. For instance, the error of a PM in three particular points, P_1, P_2 and P_3, considering the calculated error and two imaginary indices I_1 and I_2 are indicated in Table 1. Having in mind that the calculated error takes its highest value in point P_3 and the lowest one in point P_1, it is clear that index I_2 saves the order of the errors, despite index I_1, which does not follow this lead. The foregoing degeneration has a negative impact. This issue causes the curve representing the value returned by index I_4 to be delayed from the one returned by index I_1. It is apparent from Fig. 2 that the two curves do not correspond exactly to each other.

4.1.2. The Second Criterion

As a matter of fact, an applicable index should exhibit conforming reactions according to the scale of variations of the calculated error. For example, in Table 1, index I_2 is not following this rule. Even though it saves the order of the errors, but it does not provide mutations proportional to the real variations of the error in the corresponding points. Therefore, it cannot convey a meaningful representation of the efficiency of the PMs.

Moreover, as it can be observed from Fig. 3, this inadequacy results in a distortion in Fig. 3. This figure depicts an imaginary index and the calculated Cartesian workspace error of each of two imaginary PMs in three positions. For the first PM, points a_1, b_1 and c_1 are selected, for which the calculated errors are 0.7, 1 and 0.7, respectively and the prediction of the index about these values are 0.6, 1.5 and 0.6, respectively. For the second one, points a_2, b_2 and c_2 are selected, for which the calculated errors are 0.95, 1 and 0.1, respectively and the prediction of the index about these values are 0.01, 1.5 and 1.4, respectively. As it can be deduced, this index takes into account the scale of the changes of the errors.

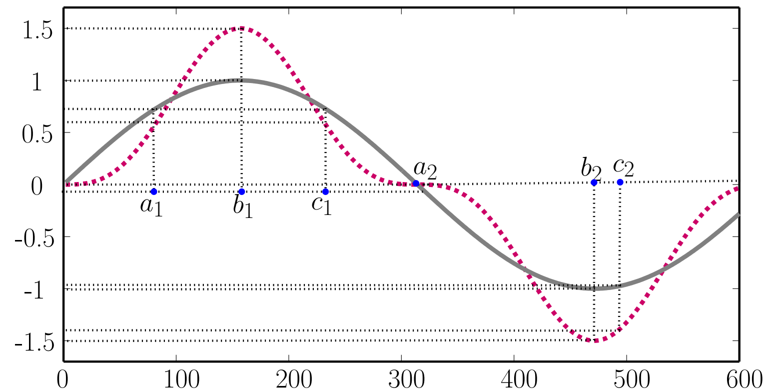

Fig. 3. The imaginary index (the dotted curve) and the calculated workspace error (the solid curve) in three positions a_1, b_1 and c_1 for the first PM and in three positions a_2, b_2 and c_2 for the second one.

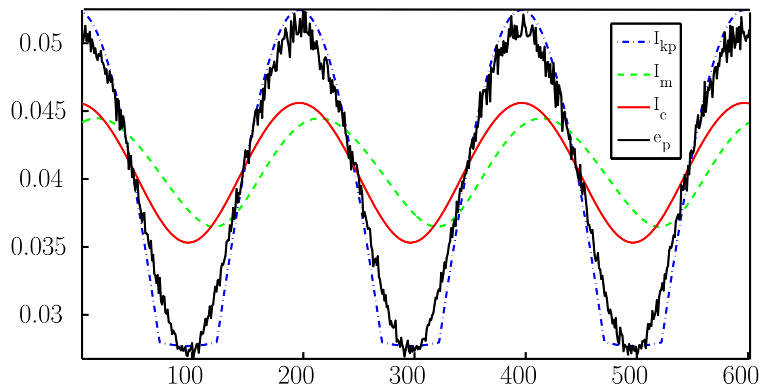

Fig. 4. Comparison of point-displacement kinematic sensitivity indices.

Usually, global error and global performance indices are considered for the sake of comparing PMs. In the above example, the global error for the first and the second PM is 2.4 and 2.05, respectively, which denotes the domination of the second one. By the way, the global index, predicts the values 2.7 and 2.91 for the PMs, respectively. Thus the index is deciding that the first PM performs better than the second one, which does not correspond to the reality. This point shows the significance importance of the criterion discussed in this section.

4.2. The Performance Indices of the 3-R$\underline{\text{P}}$R PM

In order to investigate the performance indices, the possible errors in the joint space and the Cartesian space of the 3-R$\underline{\text{P}}$R PM are simulated. To this end, some points are selected from the workspace and then the error is simulated using the kinematic and geometric relations introduced in the previous sections. To this end, generated errors are calculated in points lying on the perimeter of the circle as indicated in Fig. 1, considering a finite error in the joint space. Then, according to Eqs. (10) and (11), the indices are calculated and depicted in Figs. 4, 5 and 6. In what follows, the calculated workspace error is compared with the values returned by the other performance indices, by skipping the mathematical details.

For the sake of numerical example and better representation, consider $\phi = \pi/8.9$ which arises the most possible distance between the picks of the translational and rotational error curves, i.e., the most possible difference between the values of these indices.

In addition to the angle of rotation, the ratio of the size of the moving platform with respect to the size of

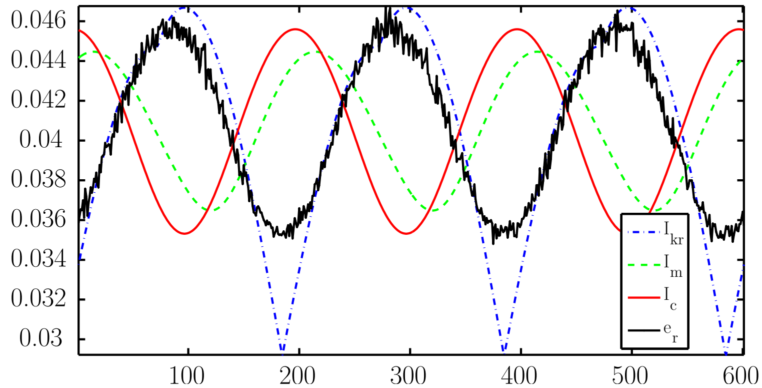

Fig. 5. Comparison of rotational kinematic sensitivity indices.

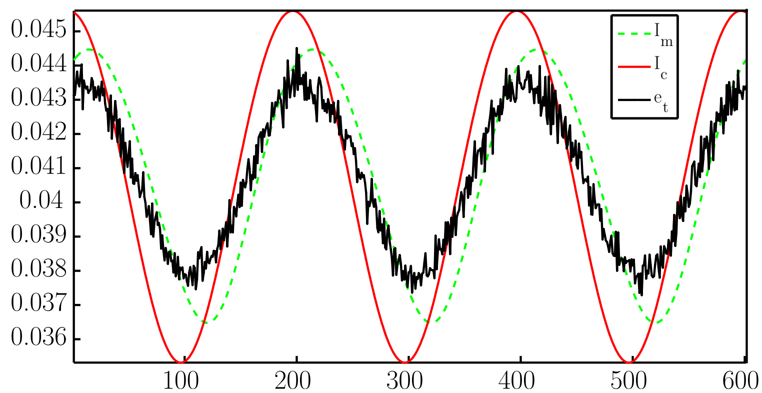

Fig. 6. Comparison of the calculated workspace error with the other performance indices.

the fixed frame, is another factor that causes the delay between the translational and rotational error curves. According to Fig. 1, consider a circle with an origin coinciding with the center of the triangle $A_1A_2A_3$ as the trajectory that the moving platform performs.

For simulating the joint space error, first the IKP is solved for the points lying on the above circle and the length of the actuated joints is obtained. Then, a random finite value is added to the above lengths as the joint space error. Subsequently, by resorting to a numerical approach the FKP is solved, and, as consequence, the pose of the moving platform is calculated. Finally, subtracting the obtained pose from the original pose of the moving platform, gives the workspace error. This action is repeated for each point lying on the perimeter of the prescribed circle represented in Fig. 1, and, finally, the global Cartesian workspace error of the PM is determined.

According to the facts that the PM is symmetric and the trajectory is circular, periodic error curve are expected with three equal periods. Thus, interpretation and comparison of the results will be more straightforward in the next steps.

5. RESULTS AND DISCUSSION

Among the components of the workspace error, two components concern point-displacement uncertainties, while the other component denotes for rotational error. It is obvious that these components are not unit-consistent and thus using the 2-norm for obtaining a single error is not reasonable.

In what follows, the error and the other indices are to be depicted and discussed. By the way, the indices are not unit-consistent and thus, cannot be compared with each other. To circumvent the latter problem, Eqs. (10) and (11) are used to adjust the scales and the offsets, respectively, for the curves of the error and the indices, which leads to bounding them within a reasonable consistent range. Subsequently, the resulted curves are depicted in Figs. 4, 5 and 6. In Fig. 4, the translational error, e_p, the maximum point-displacement kinematic sensitivity index, I_{kp}, the manipulability index, I_m, and also the condition number index using the 2-norm, I_c, are depicted. In addition, in Fig. 5, the rotational error, e_r, the maximum rotational kinematic sensitivity index, I_{kr}, the manipulability index, I_m, and also the condition number index using the 2-norm, I_c, are illustrated.

In Fig. 4, the exact coincidence of the translational error curve with the maximum point-displacement kinematic sensitivity index curve is apparent. This fact indicates the validity of the point-displacement kinematic sensitivity index, regarding each of the criteria, mentioned before. In Fig. 5, the approximate coincidence of the maximum rotational kinematic sensitivity index curve with the one for the rotational error, indicates its validity, regarding the first criterion, in spite of the other indices, whose curves show significant delays from the calculated error curve. However, the second criterion is not perfectly followed by this index, in the strictest mathematical viewpoint, and this is the cause of the little deviation of the maximum rotational kinematic sensitivity index curve from the one for the rotational error.

Albeit, according to unit-inconsistency of the translational and rotational errors, they cannot be actually compared, ignoring this fact and trusting this comparison, Fig. 6 admits that the manipulability and condition number indices are not able to authentically predict the total error, e_t.

6. CONCLUSION

This paper investigated the validity of some performance indices, such as manipulability, the condition number and the maximum point-displacement and rotational kinematic sensitivity, and also their correspondence to the reality. In fact, these indices are developed on the basis of the fact that the performance indices should react reasonably and correctly, with respect to the variations of the calculated workspace of the parallel mechanisms. Then, the validity of these indices was surveyed and compared, by applying them to the 3-R$\underline{P}$R planar parallel mechanism as case study. For this purpose, first a random finite error was considered in the joint space of the mechanism and the resulting error in the workspace of the mechanism was calculated. Moreover, the results of the indices were obtained and compared with calculated errors. The results revealed that the maximum point-displacement and maximum rotational kinematic sensitivity indices are more meaningful for the purpose of predicting the errors in the workspace of the mechanism, than the other indices, such as manipulability and the condition number. Ongoing works include extending the provided study to the all kinds of errors, being applied to the all possible trajectories of a general parallel mechanism.

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