ON THE OPTIMUM DESIGN OF PLANAR PARALLEL MECHANISMS BASED ON KINEMATIC SENSITIVITY AND WORKSPACE

Morteza Daneshmand[1], Mehdi Tale Masouleh[1], Mohammad Hossein Saadatzi[2],
Mohammad Hadi Farzaneh Kaloorazi[1] and Mohammad Bagher Menhaj[3]
[1]*Faculty of New Sciences and Technologies, University of Tehran, Tehran, Iran*
Email: mzdaneshmand@ieee.org, m.t.masouleh@ut.ac.ir, mhfarzane@ut.ac.ir
[2]*Faculty of Electrical and Computer Engineering, K. N. Toosi University of Technology, Tehran, Iran*
Email: mhsaadatzi@ieee.org
[3]*Electrical Engineering Department, Tehran Polytechnic (Amir Kabir) University of Technology, Tehran, Iran*
Email: menhaj@aut.ac.ir

ABSTRACT

This paper aims at surveying the efficiency of different types of planar parallel mechanisms, considering both kinematic sensitivity and workspace. Two performance indices are considered for the purposes of this paper, namely, point-displacement and rotational kinematic sensitivity, which are recently proposed to the end of alleviating the limitations of some notorious indices such as manipulability and dexterity. The process of refining the design parameters and finally comparing them is possible upon optimization procedures. Computational algorithms for this purpose are carried out upon the concept of genetic algorithm. Single-objective optimization is accomplished by resorting to differential evolution algorithm. For multi-objective optimization, a novel approach, referred to as *covector evolution*, is introduced, which offers the most reliable decision, in order to make a settlement between the different contradictory objectives. Finally, according to the obtained results, the performances of these mechanisms are put into contrast.

Keywords: planar parallel mechanisms; point-displacement and rotational kinematic sensitivity; single- and multi-objective optimization; differential evolution; covector evolution..

LA CONCEPTION OPTIMALE DES MÉCANISMES PARALLÈLES PLANAIRES EN FONCTION DE LA SENSIBILITÉ CINÉMATIQUE ET L'ESPACE ATTEIGNABLE

RÉSUMÉ

Cet article étudie l'efficacité des différents types de mécanismes parallèles planaires, considérant à la fois la sensibilité cinématique et l'espace atteignable. Deux indices de performance sont considérés pour les fins du présent document, la sensibilité cinématique de translation et de rotation, qui à été récemment proposé à la fin d'éliminer les limitations de certains indices notoires tels que maniabilité et la dextérité. Le processus d'affiner les paramètres de conception et enfin les comparer est possible sur des procédures d'optimisations. Les algorithmes de calcul sont basés sur le concept d'algorithme génétique. Optimisation mono-objectif est accomplie en ayant recours à l'algorithme d'évolution différentielle. Pour l'optimisation multi-objectif, une nouvelle approche, appelée *covecteur évolution*, est introduit, qui offre la décision la plus fiable, afin de faire un compromis entre différents objectifs contradictoires. Enfin, selon les résultats obtenus, les performances de ces mécanismes sont comparées.

Mots-clés : mécanismes parallèles planaires ; sensibilité cinématique de translation et de rotation ; optimisation multi-objectifs et mono-objectifs ; évolution différentielle ; évolution covecteuro.

1. INTRODUCTION

There are various efficiency indices, proposed for evaluation and comparison of Parallel Mechanisms (PMS) [1–3]. However, most of them entail several drawbacks and as a result, are not very reliable. From various studies reported in literature, it has been asserted and proved that most of the proposed kinetostatic performance indices do not indicate the accuracy of PMs properly [4, 5]. For instance, some of them consider input errors and design errors simultaneously, and are not able to distinguish them [6, 7]. In the context of kinetostatic performance indices [8], the translational and rotational parts of Jacobian matrices are not unit-consistent. As a result, the performance indices defined on the basis of this matrix do not return a physically appropriate interpretation. Therefore, defining an overall index for the performance of PMs may result in fallacious conclusions about their kinetostatic uncertainties. In what follows, the concept of Jacobian matrix is reviewed and some deficiencies of the indices defined on the basis of this non-homogeneous matrix are broadly mentioned.

The first-order kinematic relation for PMs can be developed by assuming infinitesimal variations in the input and output vectors. There are two well-known performance indices, namely dexterity [9] and manipulability [4, 10], which are defined on the basis of this matrix and consider the translational and rotational parts of Jacobian matrix simultaneously. As aforementioned, these two parts are unit-inconsistent and thus the Jacobian matrix is a non-homogeneous matrix. As a consequence of normalizing [11, 12] some unit-inconsistent variables, the performance indices defined by using this reasoning, as well as the characteristic length proposed in [13], are not physically applicable, since changing the scale of geometric properties, could change the results significantly. This assertion is proved and investigated extensively in [4, 5, 14–16].

To avoid the above inadequacy, kinematic sensitivity has been recently proposed as a unit-consistent index, where one can relied upon for comparing different PMs according to their kinematic uncertainties and making decisions on how dependable and competent they are.

A set of eight Planar Parallel Mechanisms (PPMs), taken from [17], are the subject of the study carried out in this paper, and by having in mind the reasoning given above, the kinematic sensitivity concept [18, 19] is applied to them. However, according to the classification proposed in [8, 20], this index is a posture independent measure, i.e., an index which analyzes the performance of a robot, for a predefined pose. Meanwhile, a comparison of PMs, based on a posture-dependent index, may lead to indiscriminate decision on their performance [8, 20]. At the other hand, there are posture-dependent indices, which measure the performance of the mechanical system in the whole workspace and return a meaningful and convincing value. Among these indices, the global kinematic sensitivity [21] is selected for the aim of this paper.

The optimization procedure proposed in this paper consists of two steps. First, using a single-objective procedure, namely Differential Evolution (DE) [22, 23], the process is started. This process aims at optimizing individually each of the three objectives, the point-displacement kinematic sensitivity, the rotational kinematic sensitivity and the workspace. This attitude usually results in inappropriate values, due to the fact that the algorithm devotes the other objectives to pleasing the objected one at the greatest extent.

The next step in the optimization procedure consists in using a multi-objective technique in order to approach toward a balanced state for the three conflicting objectives. There are many methods proposed and implemented in this context, such as the Pareto based approach known as Non-dominated Sorting Genetic Algorithm-II (NSGA-II) used in [16]. The foregoing method entails some deficiencies, as it necessitates the use of decision makers as to decide on the best possible solution among others, and do not return a single definite solution for the multi-objective problem. In fact, this paper follows up the study conducted in [16] by authors, by considering the aforementioned problems and circumvent them by introducing a new multi-objective concept, called *Covector Evolution Algorithm (CEA)*, which searches for the best solution among all the possibilities and suggests a definite point as one of the best one. It should be noted that some of the relations and concepts, are explained in detail in some publications by authors such as [16, 24], but

for the sake of quick references they are briefly recalled in this paper. In fact, this method is implemented and designated to find the optimal design parameters of the eight PPMs under study, by taking into account point-displacement and rotational kinematic sensitivity, together with the workspace, as the objectives of the optimization problem.
The remainder of this paper is organized as follows. First, the kinematic analysis of PPMs is briefly reviewed upon their geometric properties. The study is pursued by touching upon some fundamental concepts about applying the above kinematic traits to the kinematic sensitivity index and workspace. The single- and multi-objective optimization procedures are broadly explained where the inception is relied upon confining the design parameters to a reasonable practical range. The single-objective procedure, using DE, is applied to prepare the requisite data for the multi-objective process. Subsequently, the concept behind the proposed algorithm for the multi-objective optimization procedure, called CEA, is explained and is used to synthesis the optimal mechanism. Finally, the paper concludes with some discussions on the obtained results about the comparison of the PPMs under study in this paper.

2. REVIEW ON THE KINEMATIC MODELING AND KINEMATIC SENSITIVITY ANALYSIS OF PPMS

The mathematical framework for the formulation of the kinematic sensitivity requires a comprehensive review of the first-order kinematic relations. This point is explained in [25] and results are to be thoroughly recalled in what follows.

2.1. Kinematic Arrangement of PPMs

According to the study carried out in [17], there are ten different feasible types of PPMs. The latter PPMs with their corresponding geometrical design parameters are schematically shown in Fig. 1. However, from the study conducted in [16], it reveals that even an optimized 3-RP$\underline{\text{P}}$ and also an optimized 3-R$\underline{\text{P}}$P have a very restricted workspace, which can be practically assumed to be zero and thus these two PPMs are excluded from the rest of the study.
In Fig. 1, each architecture compromises three kinematically identical limbs. Throughout this paper, prismatic and revolute joints are denoted by P and R, respectively. In this notation, the kinematic arrangement of the joints in each leg is shown by writing the joint symbols, regarding the order from the base to the end-effector. Furthermore, the actuated joint is underlined.

2.2. Input-Output Velocity Formulation

It is worth mentioning that PMs usually may compromise various working modes. However, trajectory planning while switching between different working modes, in the most cases, is a demanding task, which regularly necessitates passing through singular regions. Thus, in this paper, just one working mode, which to the sense of kinetostatic performance leads to the best-conditioned workspace, is taken into account for each PPM, in order to obtain its input-output velocity relation, which is briefly introduced in what follows.
As it is fully described in [17], and following the same notation, the input-output velocity equation can be expressed as follows:

$$\begin{bmatrix} \boldsymbol{\zeta}_1^{\circ} \\ \boldsymbol{\zeta}_2^{\circ} \\ \boldsymbol{\zeta}_3^{\circ} \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^a \\ \dot{\theta}_2^a \\ \dot{\theta}_3^a \end{bmatrix}, \tag{1}$$

or in a matrix form:

$$\mathbf{Z}\boldsymbol{\xi} = \boldsymbol{\Lambda}\dot{\boldsymbol{\theta}} \Longrightarrow \mathbf{K}\boldsymbol{\xi} = \dot{\boldsymbol{\theta}}, \tag{2}$$

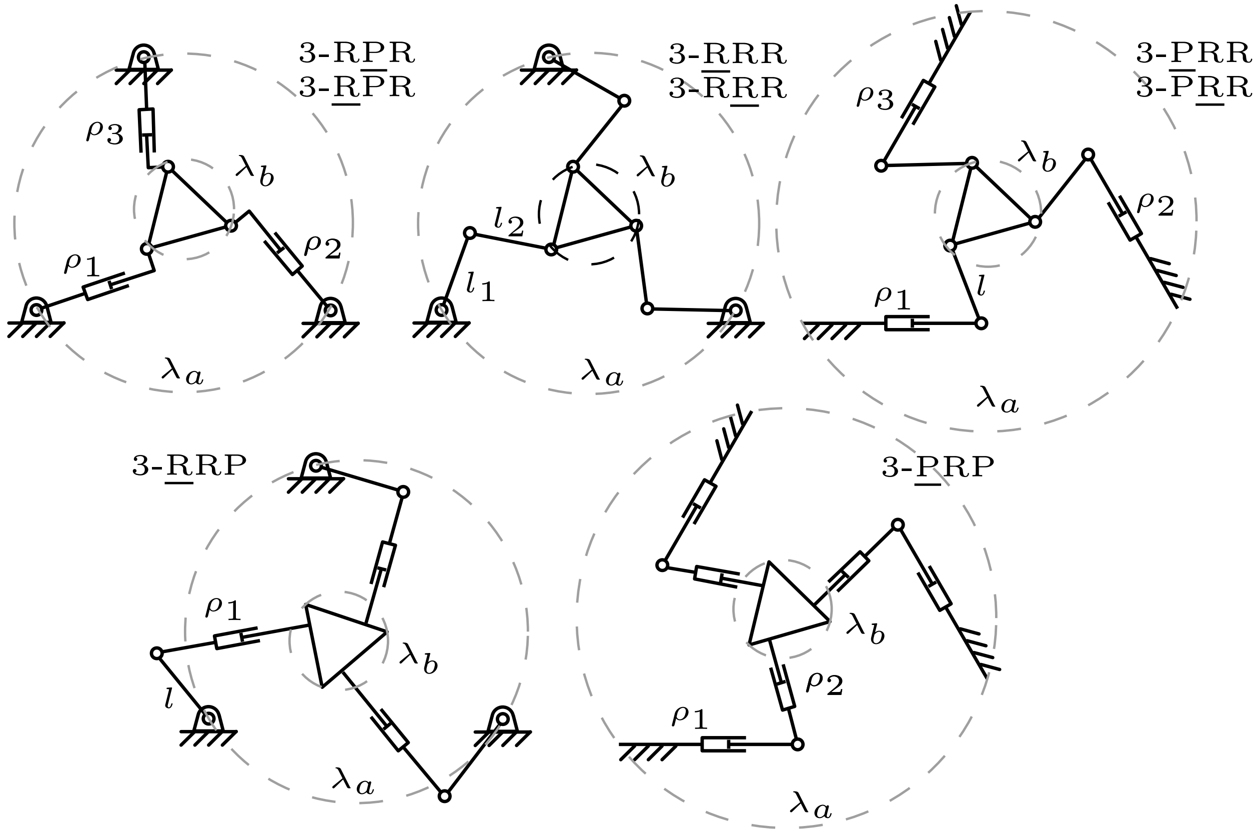

Fig. 1. 3-DOF PPMs with their corresponding geometric design parameters. The schematic is taken from [16].

where $\mathbf{K} = \mathbf{\Lambda}^{-1}\mathbf{Z}$ stands for the inverse Jacobian matrix. In the above relation, $\boldsymbol{\zeta}_i^\circ$ is the row vector of the three-dimensional matrix of wrench, implied by the i^{th} limb. Furthermore, in the above, λ_i is the moment of the reciprocal force with respect to the center of the active joint, where the actuator is revolute, or the projection of the force onto the direction of the actuated translation where the actuator is prismatic [17]. It should be noted that i is a dummy variable, which is subject to be changed throughout the paper.

2.3. Constant-orientation Workspace

The workspace of PMs is investigated under different perspectives [26]. In this paper, constant-orientation workspace is considered, which consists in the set of all possible Cartesian coordinates of a given point of the mobile platform that can be reached for a prescribed orientation [16]. The volume of the workspace can be calculated numerically using discrete integration, which can be formulated as follows [21]:

$$W = \int_{-\pi}^{\pi} A(\phi)d\phi, \tag{3}$$

where $A(\phi)$ is the area of the constant-orientation workspace.

2.4. Point-Displacement, Rotational Kinematic Sensitivity and Global Performance Indices

From the study conducted in [4], it can be inferred that geometrically, kinematic sensitivity is the maximum displacement/rotation of the moving platform under a unit displacement/rotation in the joint space. As noted previously, for a general PM, the components of the Jacobian matrix are dimensionally non-homogeneous and to overcome the latter problem, two different types of kinematic sensitivity are suggested in [4], which for a PPM can be formulated as follows:

$$\sigma_{r_{c,f}} = \max_{\|\dot{\boldsymbol{\theta}}\|_c = 1} \|\phi\|_f, \qquad \sigma_{p_{c,f}} = \max_{\|\dot{\boldsymbol{\theta}}\|_c = 1} \|\mathbf{p}\|_f, \tag{4}$$

where $\sigma_{r_{c,f}}$ and $\sigma_{p_{c,f}}$ are the rotational and point-displacement kinematic sensitivity, respectively. Furthermore, c and f are the norm of the constraints function, i.e., error in the joint space, and the norm of the pose of the moving platform, respectively. In the above, vector $\mathbf{p}$ and scalar ϕ stand for the variation of the position and orientation of the moving platform, respectively.

Two types of norms, 2- and ∞-norm, are more frequently used in practice since a geometric interpretation can be associated to them. This fact leads to four possible combinations for formulating the kinematic sensitivity. In addition, from the study reported in [5, 19], it reveals that $c = \infty$ and $f = 2$ implicit the most meaningful and reliable evaluation of the performance of the PMs, and thus are considered for the aim of this paper.

According to Eqs. (2) and (4), the constraint $\|\dot{\boldsymbol{\theta}}\|_c \leq 1$ for $c = \infty$ can be rewritten as follows:

$$\|\mathbf{Kx}\|_\infty \leq 1. \tag{5}$$

The above can be made equivalent geometrically to a polyhedron spanned by 2^n corners [4, 16, 19]. Furthermore, $\mathbf{x} = \begin{bmatrix} x & y & \phi \end{bmatrix}^T$ indicates the pose of the moving platform, i.e., the Cartesian space values of the pose of the end-effector.

The corners of the above mentioned polyhedron can be assumed as the final results of the kinametic sensitivity problem, which can be described mathematically, through a system of equations, formulated as follows:

$$\mathbf{L}\Delta\mathbf{x} \preceq \mathbf{1}_{2n}, \qquad \mathbf{L} \equiv \begin{bmatrix} \mathbf{K}^T & -\mathbf{K}^T \end{bmatrix}^T, \tag{6}$$

where the sign $\preceq$ indicates that the above inequalities should be solved component-wise. As a matter of fact, the above mentioned polyhedron is symmetric, with respect to the origin, thus one can solve just $\frac{2^n}{2}$ of the above inequalities. In the context of PPMs, there are three DOFs, i.e., $n = 3$, which implicitly leads to eight corners, leading to eight inequalities, where only four of them suffice to describe the problem of kinematic sensitivity, i.e., one has to consider four corners, namely, (x_i, y_i), $i = 1, \ldots, 4$. Thus, considering $c = \infty$ and $f = 2$ as the norms, the point-displacement kinematic sensitivity, $\sigma_{p_{\infty,2}}$, and rotational kinematic sensitivity, $\sigma_{r_{\infty,2}}$, are mathematically represented as follows [18, 19]:

$$\sigma_{p_{\infty,2}} = \max_{i=1,\ldots,4} \sqrt{x_i^2 + y_i^2}, \qquad \sigma_{r_{\infty,2}} = \max_{i=1,\ldots,4} \phi_i. \tag{7}$$

Regarding the fact that the above values are certainly nonnegative, they can be bounded to an interval from zero to one, by the following substitution:

$$\sigma'_{p_{\infty,2}} = \frac{1}{1 + \sigma_{p_{\infty,2}}}, \qquad \sigma'_{r_{\infty,2}} = \frac{1}{1 + \sigma_{r_{\infty,2}}}, \tag{8}$$

which is fully discussed in [21]. These scalar quantities convey more sensible information about the performance of the PMs. Also averaging these values in the whole workspace of the PM gives the global kinematic sensitivity [21, 27], as follows:

$$\zeta_{\sigma'_{p_{\infty,2}}} = \frac{\int_W \sigma'_{p_{\infty,2}} dW}{\int_W dW}, \qquad \zeta_{\sigma'_{r_{\infty,2}}} = \frac{\int_W \sigma'_{r_{\infty,2}} dW}{\int_W dW}. \tag{9}$$

3. SINGLE- AND MULTI-OBJECTIVE OPTIMIZATION ALGORITHMS

An optimization problem necessitates a clear and proper definition of its parameters as well as its objectives. In the case of this paper, the final goal is to lessen the effect of the kinematic uncertainties—which is equal to reducing the value of the kinematic sensitivity indices—while trying to keeping the workspace of the mechanism as great as possible. To do so, the problem can be mathematically formulated by maximizing the three scalar values obtained from Eqs. (3) and (9). Moreover, the required parameters to be varied in this procedure are the geometrical design parameters of the architectures, which are schematically shown in Fig. 1.

The eight considered PPMs are assumed to occupy an equal area in the space, so that, they can be fairly compared. To this end, their three fixed points, attached to the base, are assumed to form an equilateral triangle and lie on a circle, whose diameter being $\lambda_a = 1$. Furthermore, their design parameters should be limited to a rational range. More precisely, the stroke of actuators should be assigned within a reasonable range, which satisfies the requisitions of practical work. In addition, in order to come out with designs of practical interest, the size of the moving platform should take a reasonable value with respect to the fixed size of the base circle and all the eight considered PPMs are assumed to be constructed from three symmetric legs in each case. Therefore, only the design parameters of one leg are considered as the variables of the optimization procedure.

Generally, an optimization algorithm aims at reaching the best state for one or more of its possible objectives. However, the former usually results in unsatisfactory solutions. More clearly, the single-objective process attempts to return the best possible value for the considered objective and does not take into account the other objectives. For instance, in the context of this paper, in the most of cases, optimizing one of the kinematic sensitivity indices, leads to a very low (even sometimes bounded to only a point) workspace.

Therefore, a multi-objective process is needed to reach one or more points as possibly the most reliable solutions, which compromise the fairest balancing situation between these inconsistent objectives. To do so, the results of the single-objective algorithm play an important role, both to have an initial insight into the possible range of the objected parameters and also to survey the effect of variations of each of these parameters on the objectives of the problem.

In what follows, the above limitations are intensively reviewed. Subsequently, the concept of the single-objective procedure and its results are presented upon DE. Moreover, the CEA is elaborated, upon which, probably the best possible solutions to the optimization problem are obtained as a balancing state between the conflicting objectives. It should be noted that the insufficiency of the prepared computer algebra systems had urged the authors to develop necessary software codes, which are adapted for the purpose of this paper.

3.1. Constraints and Ranges of the Design Parameters

As stated previously, before launching the optimization process, the design parameters should be confined within an applicable range, to avoid practically unfeasible solutions. In the following, some of these constraints and ranges are shortly explained.

3.1.1. Prismatic joints

Regarding the physical limitations on the prismatic joints, they are considered to be able to take values not higher than a maximum and not lower than a minimum one, i.e., the stroke of the actuators. These physical constraints can be mathematically represented as follows:

$$\rho_{\min} > 0.1, \qquad \rho_{\max} < 0.75, \qquad \rho_{\max} > \rho_{\min}, \tag{10}$$

$$\rho_{\max} - \rho_{\min} < \rho_{\min} \Rightarrow \rho_{\max} < 2\rho_{\min}. \tag{11}$$

3.1.2. Revolute joints

These joints, regardless of the mechanical interferences, are assumed being able to take any necessary value.

3.1.3. The size of the end-effector

The three connection points, attached to the moving platform, are assumed to form an equilateral triangle and lie on a circle, whose diameter being λ_b and:

$$0.1 < \lambda_b < \lambda_a = 1. \tag{12}$$

3.1.4. Rigid links

Rigid links should satisfy practical constraints, and thus, are confined within the following range:

$$0.05 < l < 0.5. \tag{13}$$

3.2. Single-Objective Procedure, DE

The single-objective optimization is based on the concept of DE by following the reasoning proposed in [21], and the results are shown in Tables 1 and 2, taken from [16]. As mentioned before, these results bear no physically feasible meaning and are valueless for practical contexts. However, one can gain insight into the level of importance of the design parameters in affecting the objectives, and provide the required material for the multi-objective optimization, which will be explained in the next sections.

3.3. Multi-Objective Procedure, Covector Evolution

The multi-objective process, as is understandable from its virtue, should take into account all the three objectives, i.e., the point-displacement kinematic sensitivity, the rotational kinematic sensitivity and the workspace. In this paper, a novel approach is introduced, whose characteristics and its search algorithm are the central subject of this paper and are comprehensively explained in what follows.

The basic information for the multi-objective process is obtained from the results of the single-objective procedure. As the first step, the set of single-objective optimization results are rearranged as a vector, called the *intermediary covector*, as follows:

$$\mathbf{c} = \begin{bmatrix} \max\{\zeta_{\sigma'_{p_{\infty,2}}}\} & \max\{\zeta_{\sigma'_{r_{\infty,2}}}\} & \max\{W\} \end{bmatrix}^T. \tag{14}$$

The so-called intermediary covector, obviously, does not bear any physical or kinematical information, and should be arranged as a mathematical medium to ensure that the resulted set of solutions saves the proportion of the objective values and matches them to the original covector, with a little allowed deviation from the original covector. In fact, the process does not allow any objective to grow in an unreasonable scale and ruin the result of other objectives.

In this process, as it can be inferred from the nature of the single-objective process, the best desirable results are the results of the single-objective optimization. The maximum allowed deviation from the intermediary covector, for the purpose of this paper, is assumed to be 10 percent, and could be different in other contexts, according to the respective application.

The mathematical formulation for the above reasoning can be provided with a spatial imagination of the resulting set of values, being assigned in a vector aligned with the intermediary covector, or being deviated from the covector with a little angle, namely θ. In fact, $\cos\theta$ represents the percentage of correlation of the results to the desired values, and thus, in the case of this paper, should be greater than 0.9.

Objective⇒ Mechanism⇓	Parameter	$\zeta_{\sigma'_{p_\infty,2}}$	$\zeta_{\sigma'_{r_\infty,2}}$	W
3-R$\underline{\text{P}}$R	λ_b	0.4801	0.8829	0.9345
	$\rho_{\min}$	0.4228	0.2365	0.3716
	$\rho_{\max}$	0.5671	0.3791	0.7426
3-$\underline{\text{R}}$PR	λ_b	0.7726	0.9999	0.9500
	$\rho_{\min}$	0.1503	0.1732	0.3754
	$\rho_{\max}$	0.2748	0.1732	0.7499
3-R$\underline{\text{R}}$R	λ_b	0.6180	0.7474	0.2867
	l_1	0.4809	0.2780	0.5000
	l_2	0.0675	0.0691	0.5000
3-$\underline{\text{R}}$RR	λ_b	0.6728	0.7727	0.2871
	l_1	0.0614	0.0565	0.5000
	l_2	0.4325	0.2110	0.4999
3-$\underline{\text{P}}$RR	λ_b	0.6629	0.8076	0.5064
	$\rho_{\min}$	0.4372	0.1822	0.2427
	$\rho_{\max}$	0.7130	0.3345	0.4842
	l	0.4414	0.2084	0.4376
3-P$\underline{\text{R}}$R	λ_b	0.5981	0.6843	0.5036
	$\rho_{\min}$	0.3370	0.5010	0.2432
	$\rho_{\max}$	0.6117	0.7500	0.4850
	l	0.0734	0.0655	0.4386
3-$\underline{\text{R}}$RP	λ_b	0.7284	0.7283	0.1369
	$\rho_{\min}$	0.3571	0.3571	0.2037
	$\rho_{\max}$	0.7142	0.7141	0.4061
	l	0.3712	0.3712	0.4453
3-$\underline{\text{P}}$RP	λ_b	0.2000	0.1000	0.9784
	$\rho_{1\min}$	0.2151	0.2091	0.3746
	$\rho_{1\max}$	0.3933	0.4014	0.7478
	$\rho_{2\min}$	0.2704	0.2436	0.3351
	$\rho_{2\max}$	0.5010	0.4308	0.6693

Table 1. The results of the single-objective optimization procedure for selecting the best design parameters. The table is taken from [16].

Finally, the cost-function to be maximized for the multi-objective optimization can be defined as follows:

$$j = \frac{\mathbf{c}^T \begin{bmatrix} \zeta_{\sigma'_{p_\infty,2}} & \zeta_{\sigma'_{r_\infty,2}} & W \end{bmatrix}^T}{\|\mathbf{c}\|^2}, \qquad \text{s.t} \qquad \cos\theta \geq 0.9, \tag{15}$$

which implies that the solution to the multi-objective problem seeks the farthest possible point on the intermediary covector, by taking into account its orientation, i.e., the solution should be oriented in a direction almost the same as the intermediary covector, unless it deviates from the intermediary covector with a small negligible angel. The above mentioned process is applied to the all eight PPMs represented in Fig. 1 and the results are shown in Tables 3 and 4, for the PPMs with prismatic and revolute actuators, respectively. It should be noted that, in these tables, the parameter S represents the respective length of the resulted vector,

Objective⇒ Mechanism⇓	Result	$\zeta_{\sigma'_{p_{\infty},2}}$	$\zeta_{\sigma'_{r_{\infty},2}}$	W
3-R$\underline{\text{P}}$R	$\zeta_{\sigma'_{p_{\infty},2}}$	0.4060	0.3411	0.0767
	$\zeta_{\sigma'_{r_{\infty},2}}$	0.1896	0.2984	0.0880
	W	0.0015	0.0015	1.2140
3-$\underline{\text{R}}$PR	$\zeta_{\sigma'_{p_{\infty},2}}$	0.7639	0.0006	0.0835
	$\zeta_{\sigma'_{r_{\infty},2}}$	0.5198	0.7143	0.2942
	W	0.0015	0.0089	1.2199
3-R$\underline{\text{R}}$R	$\zeta_{\sigma'_{p_{\infty},2}}$	0.9999	0.9981	0.3351
	$\zeta_{\sigma'_{r_{\infty},2}}$	0.9999	0.9975	0.1061
	W	0.0015	0.0015	6.8974
3-$\underline{\text{R}}$RR	$\zeta_{\sigma'_{p_{\infty},2}}$	0.9927	0.9855	0.3272
	$\zeta_{\sigma'_{r_{\infty},2}}$	0.9898	0.9980	0.1171
	W	0.0015	0.0015	6.8974
3-$\underline{\text{P}}$RR	$\zeta_{\sigma'_{p_{\infty},2}}$	0.8856	0.5878	0.1697
	$\zeta_{\sigma'_{r_{\infty},2}}$	0.3190	0.5116	0.1191
	W	0.0089	0.0044	0.2739
3-P$\underline{\text{R}}$R	$\zeta_{\sigma'_{p_{\infty},2}}$	1.0000	0.9860	0.3659
	$\zeta_{\sigma'_{r_{\infty},2}}$	0.8680	0.9688	0.1447
	W	0.0015	0.0015	0.2739
3-$\underline{\text{R}}$RP	$\zeta_{\sigma'_{p_{\infty},2}}$	0.7260	0.7260	0.2948
	$\zeta_{\sigma'_{r_{\infty},2}}$	0.5759	0.5759	0.1566
	W	0.0030	0.0030	0.1318
3-$\underline{\text{P}}$RP	$\zeta_{\sigma'_{p_{\infty},2}}$	0.4797	0.4550	0.3227
	$\zeta_{\sigma'_{r_{\infty},2}}$	0.2626	0.2695	0.1949
	W	0.0015	0.0015	0.1125

Table 2. The results of each objective, using the values obtained from Table 1. The table is taken from [16].

with regard to the length of the original covector.

3.3.1. Results and Discussion

The bottom line in analyzing the results of the multi-objective algorithm is that the compared values should follow the unit-consistency rule. By applying the above reasoning, it is apparent that the point-displacement kinematic sensitivity of the PPMs with prismatic actuators is unit-less, while the unit of their rotational kinematic sensitivity is $\frac{\text{rad}}{\text{m}}$. In turn, for PPMs with revolute actuators the unit of point-displacement kinematic sensitivity is $\frac{\text{m}}{\text{rad}}$ and their rotational kinematic sensitivity is unit-less. This fact necessitates a task of reclassification of the PPMs into two sets, three with prismatic actuators and the other five with revolute actuators.

- Mechanisms with prismatic actuators: As it can be inferred from the Table 3, among these mechanisms, the 3-$\underline{\text{P}}$RR exhibits the best performance according to both point-displacement and rotational

Result⇒ Mechanism⇓	Parameters					$\zeta_{\sigma'_{p_\infty,2}}$	$\zeta_{\sigma'_{r_\infty,2}}$	W	S
3-R$\underline{\text{P}}$R	λ_b 0.9540	$\rho_{\min}$ 0.3751	$\rho_{\max}$ 0.7500	-	-	0.0768	0.0900	1.2199	0.9324
3-$\underline{\text{P}}$RR	λ_b 0.6443	$\rho_{\min}$ 0.4223	$\rho_{\max}$ 0.7132	l 0.3676	-	0.8742	0.3124	0.0074	0.8768
3-$\underline{\text{P}}$RP	λ_b 0.1000	$\rho_{1\min}$ 0.4720	$\rho_{1\max}$ 0.6768	$\rho_{2\min}$ 0.1910	$\rho_{2\max}$ 0.3766	0.4651	0.2695	0.0015	0.9572

Table 3. The results of the multi-objective optimization, utilizing covector evolution algorithm for the PPMs with prismatic actuators.

Result⇒ Mechanism⇓	Parameters				$\zeta_{\sigma'_{p_\infty,2}}$	$\zeta_{\sigma'_{r_\infty,2}}$	W	S
3-$\underline{\text{R}}$PR	λ_b 0.8840	$\rho_{\min}$ 0.3750	$\rho_{\max}$ 0.7498	-	0.1480	0.3089	1.1562	0.7505
3-R$\underline{\text{R}}$R	λ_b 0.2977	l_1 0.5000	l_2 0.5000	-	0.3361	0.1100	6.9003	0.9813
3-$\underline{\text{R}}$RR	λ_b 0.2273	l_1 0.4997	l_2 0.5000	-	0.3334	0.1001	6.8974	0.9810
3-P$\underline{\text{R}}$R	λ_b 0.6274	$\rho_{\min}$ 0.4705	$\rho_{\max}$ 0.7407	l 0.0650	0.9988	0.9973	0.0015	0.9947
3-$\underline{\text{R}}$RP	λ_b 0.7474	$\rho_{\min}$ 0.3653	$\rho_{\max}$ 0.7219	l 0.3685	0.7183	0.5700	0.0030	0.9796

Table 4. The results of the multi-objective optimization, utilizing covector evolution algorithm for the PPMs with revolute actuators.

kinematic sensitivity. Furthermore, the 3-R$\underline{\text{P}}$R has the best workspace.

- Mechanisms with revolute actuators: As it can be inferred from the Table 4, among these mechanisms, the 3-P$\underline{\text{R}}$R exhibits the best performance according to both point-displacement and rotational kinematic sensitivity. Furthermore, the 3-R$\underline{\text{R}}$R and 3-$\underline{\text{R}}$RR have the best workspace.

4. CONCLUSION

This paper surveyed the kinetostatic performance and also the workspace optimization of eight planar parallel mechanisms. Geometrical design parameters of these architectures were subject to be optimized with respect to three main goals, including point-displacement kinematic sensitivity, rotational kinematic sensitivity and workspace. As the first step of the optimization procedure, a single-objective algorithm was applied, using the concept of an evolutionary algorithm, called differential evolution. This process aimed at obtaining an initial insight to the effect of variations of each design parameter on each objective, and also provided the necessary material for the next step, i.e., the multi-objective optimization. Finally, a multi-objective optimization was introduced, called the *covector evolution algorithm*. The results of the proposed formulation for the multi-objective optimization revealed that among mechanisms with prismatic actuators, the 3-$\underline{\text{P}}$RR exhibits the best performance, according to both point-displacement and rotational kinematic sensitivity, and the 3-R$\underline{\text{P}}$R has the best workspace. However, it was inferred that among mechanisms with revolute actuators, the 3-P$\underline{\text{R}}$R exhibits the best performance, according to both point-displacement and rotational kinematic sensitivity, and the 3-R$\underline{\text{R}}$R and 3-$\underline{\text{R}}$RR have the best workspace, a result which was sought to be obtained at the outset. Furthermore, according to the fact that the optimization procedures have been implemented considering the kinematic sensitivity as one of the criteria, they have contributed to lessen the area of the singular regions in the workspace of the planar parallel mechanisms. By the way, the proposed designs, more or less, suffer from singularities, which excludes the proposed optimization procedures and resulted sets of design parameters from immediate practical applicability. Therefore, inevitably, the determination of the largest singularity-free ellipsoid in the workspace of the planar parallel mechanisms and considering it as the workspace index should be included in the future work.

REFERENCES

1. Caro, S., Bennis, F. and Wenger, P. "Tolerance Synthesis of Mechanisms: A Robust Design Approach." *Journal of Mechanical Design*, Vol. 127, pp. 86–94, 2005.
2. Binaud, N., Caro, S. and Wenger, P. "Sensitivity Comparison of Planar Parallel Manipulators." *Mechanism and Machine Theory*, Vol. 45, No. 11, pp. 1477–1490, 2010.
3. Daneshmand, M., Saadatzi, M. H., Tale Masouleh, M. and Menhaj, M. B. "Optimization of Kinematic Sensitivity and Workspace of Planar Parallel Mechanisms." In "Accepted for Multibody Dynamics Thematic Conference," Zagreb, Croatia, Jul. 2013.
4. Cardou, P., Bouchard, S. and Gosselin, C. "Kinematic-sensitivity Indices for Dimensionally Nonhomogeneous Jacobian Matrices." *IEEE Transactions on Robotics*, Vol. 26, No. 1, pp. 166–173, 2010.
5. Merlet, J. "Jacobian, Manipulability, Condition Number, and Accuracy of Parallel Robots." *Journal of Mechanical Design*, Vol. 128, pp. 199–206, 2006.
6. Briot, S. and Bonev, I. "Are Parallel Robots More Accurate than Serial Robots?" *Transactions of the Canadian Society for Mechanical Engineering*, Vol. 31, No. 4, pp. 445–456, 2007.
7. Wenger, P., Gosselin, C. and Maillé, B. "A Comparative Study of Serial and Parallel Mechanism Topologies for Machine Tools.", 1999.
8. Angeles, J. *Fundamentals of Robotic Mechanical Systems: Theory, Methods, and Algorithms*. Springer, 2006.
9. Gosselin, C. M. "The Optimum Design of Robotic Manipulators Using Dexterity Indices." *Robotics and Autonomous Systems*, Vol. 9, No. 4, pp. 213–226, 1992.
10. Yoshikawa, T. "Analysis and Control of Robot Manipulators with Redundancy." In "Robotics Research: The First International Symposium," pp. 735–747. MIT Press Cambridge, MA, 1984.
11. Angeles, J. and Ma, O. "Performance Evaluation of Four-bar Linkages For Rigid-body Guidance Based on Generalized Coupler Curves." *Journal of Mechanical Design*, Vol. 113, pp. 17–24, 1991.
12. Angeles, J. "The Design of Isotropic Manipulator Architectures in the Presence of Redundancies." *The International Journal of Robotics Research*, Vol. 11, No. 3, pp. 196–201, 1992.
13. Angeles, J. "Is there a Characteristic Length of a Rigid-body Displacement?" *Mechanism and Machine Theory*, Vol. 41, No. 8, pp. 884–896, 2006.
14. Yoshikawa, T. "Manipulability of Robotic Mechanisms." *The International Journal of Robotics Research*, Vol. 4, No. 2, pp. 3–9, 1985.
15. Stocco, L. J., Salcudean, S. and Sassani, F. "On the Use of Scaling Matrices for Task-specific Robot Design." *IEEE Transactions on Robotics and Automation*, Vol. 15, No. 5, pp. 958–965, 1999.
16. Daneshmand, M., Saadatzi, M. H. and Tale Masouleh, M. "Kinematic Sensitivity and Workspace Optimization of Planar Parallel Mechanisms Using Evolutionary Techniques." In "First International Conference on Robotics and Mechatronics (ICRoM)," IEEE, Tehran, Iran, Feb. 2013.
17. Bonev, I. A., Zlatanov, D. and Gosselin, C. "Singularity Analysis of 3-DOF Planar Parallel Mechanisms via Screw Theory." *Journal of Mechanical Design*, Vol. 125, No. 3, pp. 573 – 581, Sep. 2003.
18. Saadatzi, M. H. *Workspace and Singularity Analysis of 5–DOF Symmetrical Parallel Robots with Linear Actuators*. Master's thesis, Faculty of Electrical and Computer Engineering, K.N. Toosi University of Technology, Tehran, Iran, Summer. 2011.
19. Saadatzi, M. H., Tale Masouleh, M., Taghirad, H.D., Gosselin, C. and Cardou, P. "Geometric Analysis of the Kinematic Sensitivity of Planar Parallel Mechanisms." *Transactions of the Canadian Society for Mechanical Engineering*, Vol. 35, No. 4, pp. 477–490, Jun. 2011.

20. Khan, W. A. and Angeles, J. “The Kinetostatic Optimization of Robotic Manipulators: The Inverse and the Direct Problems.” *Journal of Mechanical Design*, Vol. 128, No. 1, pp. 168–178, 2006.
21. Saadatzi, M. H., Tale Masouleh, M., Taghirad, H. D., Gosselin, C. and Teshnehlab, M. “Multi-Objective Scale Independent Optimization of 3-RPR Parallel Mechanisms.” In “13th World Congress in Mechanism and Machine Science,” Guanajuato, Mexico, Jun. 2011.
22. Storn, R. and Price, K. “Differential Evolution - A Simple and Efficient Heuristic for Global Optimization over Continuous Spaces.” *Journal of Global Optimization*, Vol. 11, pp. 341 – 359, Dec. 1997.
23. Price, K. V., Storn, R. M. and Lampinen, J. A. *Differential Evolution: A Practical Approach to Global Optimization*. 1st ed.. Springer-Verlag, New York, 2005.
24. Saadatzi, M. H., Tale Masouleh, M., Taghirad, H. D., Gosselin, C. and Cardou, P. “Geometric Analysis of the Kinematic Sensitivity of Planar Parallel Mechanisms.” In “CCToMM Conference,” , Jun. 2011.
25. Bonev, I. A. *Geometric Analysis of Parallel Mechanisms*. Ph.D. thesis, Laval University, Quebec, Canada, Nov. 2002.
26. GOSSELIN, C. “Determination of the Workspace of 6-DOF Parallel Manipulators.” *Journal of Mechanical Design*, Vol. 112, No. 3, pp. 331–336, 1990.
27. Saadatzi, M. H., Tale Masouleh, M., Taghirad, H. D., Gosselin, C. and Cardou, P. “On the Optimum Design of 3-R$\underline{P}$R Parallel Mechanisms.” In “19th Iranian Conference on Electrical Engineering (ICEE),” pp. 1–6. IEEE, 2011.