Dynamic Analysis and Control of Cable Driven Robots Considering Elasticity in Cables

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Abstract
In this paper modeling and control of cable-driven redundant parallel manipulators with flexible cables are studied in detail. In this class of robots cables should remain in tension in the whole workspace. Based on new results, in fully constraint cable robots, cables can be modeled as axial springs. With this assumption the system is modeled using Lagrange's formulation. Then, control algorithm is proposed. Internal forces are introduced and incorporated in the proposed control algorithm. This algorithm is formed in cable length coordinates in which the internal forces play important role. Finally, the closed loop system is proved to be asymptotically stable, through Lyapunov analysis, and the performance of the proposed algorithm is studied through simulation.

Keywords: Cable driven robot, Redundancy, Modeling, Vibration, Lyapunov stability.

1 INTRODUCTION

Cable driven robots are a class of parallel robots in which the rigid links are replaced by cables. A cable driven robot consists of a moving end-effector and a number of active cables connected to the end-effector. These cables are fixed on the base with actuating motors and pulleys. While the cable length is changing, the position and orientation of the end-effector is pulled toward its desired values. Cable driven robots have some advantages compared to conventional robots. Because of using cables instead of rigid links, they have the great potential to use in very large workspace applications such as large adaptive reflector and SkyCam [1, 2]. Since cables have negligible mass and inertia, this type of robots are suitable for high acceleration applications. Moreover, they can achieve some useful characteristics such as "Transportability and ease of assembly/disassembly", "Reconfigurability by changing the location of motors and updating the control algorithm" and "Economical structure and maintenance due to simple mechanical structure and using low cost and simple mechanical components". Consequently, cable driven robots are exceptionally suitable to be used in many applications such as, handling of heavy materials [3], high speed manipulation

[4], rapidly deployable rescue robots, cleanup of disaster areas [5], and access to remote locations and interaction with hazardous environment [6].

The most important limitation of cable driven robots is that, the cables suffer from unidirectional constraints that can only pull and not push, while general parallel robots have actuators that can provide bidirectional tension. In this class of robots, the cables must be in tension in the whole workspace. Based on this fact, cable driven robots can be classified into two types; under constrained and fully constrained [5]. This paper is concerned about fully constrained cable driven robots. A major challenge in mechanical and control design of this class of robots, is the nonlinear behavior of the cables. Cables are usually flexible and have to encounter some unavoidable situations such as elongation because of the cable driven characters. This flexibility may lead to position and orientation errors. Moreover, the system may lead to vibration, and cause the whole system to be uncontrollable. Vibration of a cable driven robot may be a concern for some applications which require high bandwidth or high stiffness of the system [7]. Though, the cable characteristics have been studied from long time ago, especially in civil engineering, using cables in parallel robots demonstrates a quite new application. Generally, in civil engineering cables are heavy and bulky materials, whose static analysis is studied in order to stabilize the bridge type structures [8, 9]. However, in cable driven robots the cables are very light, and the motion of the end-effector is generated by change of length of the cables. Because of this new application and research trend, reported studies on the effect of cable flexibility on modeling, optimal design and control of such manipulators are very limited and usually neglected.

Stiffness analysis of the cable robots with flexible cables, may be seen in Behzadipour's and Kozak's works [10, 11]. Behzadipour introduced a four springs model for cable and achieves necessary and sufficient conditions for stability of system based on positive definiteness of the robot stiffness matrix [10]. Kozak in [11], considers the mass of the cables and by using a static model of cables shows how cable sagging affects the kinematics and stiffness of the system. In [12], a static model of cable is proposed and static deformation of cables is achieved. The vibration problem of cable driven robots has received less attention. To model the vibration due to flexibility of cables, Agrawal has used wave equation to model the cable vibration, providing the cables length are constant [13]. However in practice this assumption is not true for cable driven robots. Kawamura et al. showed that the internal forces of cables can play a vital role in reducing the vibration of the system [14]. To investigate the vibration analysis of cable driven robots, linear and nonlinear springs are used to model the behavior of cables [12, 14]. But in all reported research, it is assumed that cable has only axial flexibility and the transversal flexibility is ignored.

In the dynamic modeling of cable driven robots, this point should be noticed that a complete dynamic model of cable robots is very complicated. Furthermore, since the obtained model should be used in controller design strategies, such complicated models are useless for this objective, although they can accurately describe dynamic intrinsic characteristics of cables. Thus, in practice it is proposed to include only the dominant effects in the dynamics analysis. For this reason in many robotics applications, cables mass have been neglected and cable has been considered as a rigid element [15, 16]. With those assumptions the dynamics of cable driven robot is reduced to the end-effector dynamics. However, in practice using this assumption will lead to some inaccuracies in control especially the stability of the manipulator. In this paper a more precise model of the cable driven robot considering cable flexibility is derived and being used in the controller design and stability analysis. Using natural frequencies of system, Diao and Ma have shown in [7], that in fully constrained cable driven robots the vibration of cable manipulator due to the transversal vibration of cables can be ignored in comparison to that of cable axial flexibility. In other words, it has been justified to just model the cable as an axial spring in cable driven robots. By this means, this model can describe the dominant dynamic characteristics of cable and can be used in the dynamic model of cable robot. Based on this observation, in this paper axial spring is used to model cable dynamics.

In this paper, considering axial flexibility in cables, a new dynamic model for fully constrained cable driven robots is presented. In this structure the cables lengths with and without tension are considered as the describing states in the model. By using the obtained model, the control of the system is studied, while keeping the cables in tension is a critical point, which is well addressed in the proposed control strategy of the system. Next, the stability of the system is analyzed through Lyapunov second method, and it is proven that the closed–loop system with the proposed control algorithm is stable. Finally the performance of the proposed algorithm is examined through simulation.

2 DYNAMICS ANALYSIS

2.1 Robot Dynamics with Ideal Cables

In this section let us first assume that the mass and the flexibility of the cables can be ignored, since they are much smaller and lighter than other mechanical parts. With this assumption the dynamics of cable driven robot reduces to that of the end-effector. Therefore, the dynamics of system can be expressed by the following vector equation [15, 16]:

$$M(x)\ddot{x} + N(x,\dot{x}) = J^T\tau \tag{1}$$

in which,

$$N(x,\dot{x}) = C(x,\dot{x})\dot{x} + G(x) \tag{2}$$

and,
$M(x)$: Mass matrix of the system, $C(x,\dot{x})\dot{x}$: Coriolis and centrifugal terms, $G(x)$: Vector of gravity terms, J: Jacobian matrix of system and x: Vector of generalized coordinates.
On the other hand, the actuators dynamics is represented by

$$I\ddot{q} + D\dot{q} + r\tau = u \tag{3}$$

Where,
q: Angles vector of motors shaft, I: Actuator moments of inertia matrix, D: Actuator viscous friction matrix, r: The radius of drums, τ: Cable tension vector and u: Motor torque vector.
As for the position reference, define all q to be zero when the end-effector centroid is located at the center of the frame; from this configuration positive angle q will cause a change ΔL in cable lengths, therefore, we have:

$$rq = \Delta L = L - L_0 \Longrightarrow q = r^{-1}(L - L_0) \tag{4}$$

By differentiating and using manipulator Jacobian definition $L = J\dot{x}$:

$$\dot{q} = r^{-1}\dot{L} = r^{-1}J\dot{x} \ , \ \ \ddot{q} = r^{-1}J\ddot{x} + r^{-1}\dot{J}\dot{x} \tag{5}$$

Using equations (5), (3) and (1) and some manipulations we can show that:

$$M_{eq}(x)\ddot{x} + N_{eq}(x,\dot{x}) = J^T u \tag{6}$$

In which,

$$\begin{aligned} M_{eq} &= rM(x) + r^{-1}J^T I J \\ N_{eq} &= rN(x,\dot{x}) + r^{-1}J^T I \dot{J}\dot{x} + r^{-1}J^T D J\dot{x} \end{aligned} \tag{7}$$

It can be seen that actuator dynamics is transferred to Cartesian space by Jacobian matrix, which is a projection from length space to Cartesian space.

2.2 Robot Dynamics with Real Cables

In the parallel cable driven robots, each cable is rolled by a pully which is driven by an electric motor with gears. As it is said before, in order to remedy the vibration problem, elasticity of cables must be considered [7, 14]. In this type of robots when the flexibility in cables is considered, actuator position (motor rotation for opening the cable) is not directly related to end-effector position. Thus for such models, position of actuators and position of end-effector may be considered as vector of system states. In other words, both the cable lengths before tension and the cable lengths after tension (that are related to end-effector position by Jacobian matrix), may be used as independent variables in order to analyze the dynamics of the manipulator. New research results have shown that in fully constrained cable robots, dominant dynamics of cables are longitudinal vibration [7], therefore, axial spring model can suitably describe the effects of dominant dynamics of cable.

In order to model a general cable driven robot with n cables assume that: $L_{1i} : i = 1, 2, ..., n$ denotes the length of i^{th} cable with tension and $L_{2i} : i = 1, 2, ..., n$ denotes the length of the i^{th} cable without tension. If the system is rigid, then $L_{1i} = L_{2i}$, $\forall i$.

$$L = (L_{11}, L_{12}, \cdots, L_{1n}, L_{21}, L_{22}, \cdots, L_{2n}) = (L_1^T, L_2^T) \tag{8}$$

Furthermore, if the flexibility is modeled with a linear axial spring with constant k_i, then the potential energy of system can be expressed by: $P = P_0 + P_1$. In this equation P_0 is the potential energy of rigid robot and P_1 is the potential energy of cable. Using linear axial spring model for cable, the total potential energy of a cable is

$$P_1 = \frac{1}{2}(L_1 - L_2)^T K (L_1 - L_2) \tag{9}$$

where, K is the stiffness matrix of cables. Kinetic energy of system is

$$K = \frac{1}{2}\dot{x}^T M(x)\dot{x} + \frac{1}{2}\dot{q}^T I_m \dot{q} \tag{10}$$

In which x is the generalized coordinate in cartesian space, q is the motor shaft position vector, $M(x)$ is the mass matrix and I_m is the actuator moments of inertia. The lagrangian is expressed by

$$\mathcal{L} = K - P = \frac{1}{2}\dot{x}^T M(x)\dot{x} + \frac{1}{2}\dot{q}^T I_m \dot{q} - P_0 - \frac{1}{2}(L_1 - L_2)^T K (L_1 - L_2) \tag{11}$$

The equations of motion of the system can be written using Euler-Lagrange formulation:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{y}_i}\right) - \frac{\partial \mathcal{L}}{\partial y_i} = Q_i \tag{12}$$

The final equation of motion can be written in the following form:

$$M(x)\ddot{x} + N(x, \dot{x}) = J^T K(L_2 - L_1) \tag{13}$$

$$I_m \ddot{q} + rK(L_2 - L_1) + D\dot{q} = u \tag{14}$$

in which,

$$N(x, \dot{x}) = C(x, \dot{x})\dot{x} + G(x) \ , \ \ L_2 - L_0 = rq$$

In these equations $\dot{L}_1 = J\dot{x}$, and furthermore:
x: Vector of generalized coordinates, q: Vector of angles of motors shaft, K: Stiffness matrix of cables, J: Jacobian matrix, $M(x)$: Mass matrix of the rigid system, $C(x, \dot{x})\dot{x}$: Coriolis and centrifugal terms, $G(x)$: Vector of gravity terms, I_m: Actuator moments of inertia matrix, D: Actuator viscous friction matrix, r: The radius of drums and u: Motor torque vector.
Equations (13) and (14) represent cable driven robot as a nonlinear and coupled system. This representation includes both rigid and flexible subsystems and their interactions.

3 Controller Design

3.1 Internal Forces

In cable driven robot end-effector is suspended by cables. However, since cables can provide only tension, actuator redundancy is necessary due to unidirectionality of cable tension. Such redundant actuation for a cable driven robot is similar to the actuation of the multi-fingered robots of which contact between fingers and an object is regarded as frictionless points. Therefore, it is possible to apply the concept of "Vector Closure" which was mentioned in the research of multi-finger robots to parallel cable driven robots. Generally, Vector Closure is expressed in the following way [17]:
In an n-dimensional space, a set of vector J^T is a vector closure if and only if J^T has a least $n+1$ vectors $(j_1 ... j_{n+1})$ satisfying the following two conditions:
1) Each set of n vectors in $n+1$ vectors is linearly independent.
2) A vector $\beta = (\beta_1 ... \beta_{n+1})^T$ exists, that satisfies

$$\sum_{i=1}^{n+1} j_i \beta_i = 0_{n\times 1} \tag{15}$$

in which, each element of the vector β has the same sign (positive or negative),
$\beta_i > 0$ (for any i) or $\beta_i < 0$ (for any i).

It is well known in parallel robots that Jacobian transpose relates the resultant forces f applied on the end-effector to the cable tensions τ [18]:

$$f = J^T \tau \tag{16}$$

where the matrix J^T denotes Jacobian transpose matrix and may be expressed by its columns as : $J^T = [j_1, j_2, ..., j_m]$. The Vector closure conditions mean that each cable tension remains positive and any resultant force vector can be generated. This result implies that at least $n + 1$ cables are necessary to realize the motion with n degrees of freedom. Since in cable driven robots actuator redundancy is a requirement, the number of the cable actuators are greater than the degrees of freedom, and therefore, the Jacobian matrix is not square. The inverse relation to calculate the tension in cables from the resultant force using pseudo inverse may be given by:

$$\tau = J^{\dagger} f + (I - J^{\dagger} J^T) c \tag{17}$$

In which, $J^{\dagger}$ denotes the pseudo inverse of the matrix J^T. The second term is generated from the null space of J^T which can be considered as internal forces among the cables. Notice that since the internal force lies in the null space of J^T it does not contribute into the driving force to the end-effector, and it only produces tension in cables in order to keep all the cables in tension. Internal force plays an important role in our proposed control algorithm.

3.2 Control Algorithm in Cable Length Space

In this section proposed control algorithm in cable length space is discussed. In this algorithm we use internal forces to ensure that all cables are in tension. A cable driven robot must satisfy this condition in its whole workspace. By using desired set point vector q_d, the control input u is proposed to be:

$$u = K_p(q_d - q) - K_v \dot{q} + Q + rQ_G + K_s(\dot{L}_2 - \dot{L}_1) \tag{18}$$

where, $K_p(n \times n)$ [n is the number of cables], $K_v(n \times n)$ and $K_s(n \times n))$ denote feedback gain matrices. The term $Q(n \times n)$ denote internal force vector and satisfy

$$J^T Q = 0 \tag{19}$$

It is important to note that the vector Q does not contribute into motion of the end-effector, and only causes internal forces in the cables. This term ensures that all cables remain in tension in the whole workspace. Moreover, this term increases the stiffness of the system, and as a result, minimizes the vibration in transversal direction of the cables. The term Q_G is added to compensate the gravitational force. This vector must satisfy

$$J^T Q_G = G(x) \tag{20}$$

Furtheremore, L_2 and L_1 vectors are cable lengths without and with tension, respectively. L_2 is measured by shaft encoder in the side of motor and related to q by $rq = L_2 - L_0$. L_1 show cable lengths when cables are in tension and is measured by pot strings. In the following section we discuss the stability of the closed–loop system based on this proposed control algorithm.

3.3 Stability Analysis

To show that the control law given in equation (18) achieves set point tracking, consider the following Lyapunov function:

$$V = \frac{1}{2}\dot{q}^T I_m \dot{q} + \frac{1}{2}\dot{x}^T M(x)\dot{x} + \frac{1}{2}(L_1 - L_2)^T K(L_1 - L_2) + \frac{1}{2}(q_d - q)^T K_p(q_d - q) \tag{21}$$

The Lyapunov function is generated using the total energy in the system. Since q_d is constant, the time derivative of the Lyapunov function V is given by:

$$\dot{V} = \dot{q}^T I_m \ddot{q} + \dot{x}^T M(x)\ddot{x} + \frac{1}{2}\dot{x}^T \dot{M}(x)\dot{x} + (L_1 - L_2)^T K(\dot{L}_1 - \dot{L}_2) + (q_d - q)^T K_p(-\dot{q}) \tag{22}$$

Substitute equation (13) and (14) and proposed control effort (18) in (22) :

$$\begin{gathered}\dot{V} = \dot{q}^T \left[K_p(q_d - q) - K_v\dot{q} + Q + rQ_G + K_s(\dot{L}_2 - \dot{L}_1)\right] - \dot{q}^T D\dot{q} - \dot{q}^T rK(L_2 - L_1) + \\ \dot{x}^T \left[J^T K(L_2 - L_1) - C(x,\dot{x})\dot{x} - G(x)\right] + \tfrac{1}{2}\dot{x}^T \dot{M}(x)\dot{x} \\ +(L_1 - L_2)^T K(\dot{L}_1 - \dot{L}_2) - (q_d - q)^T K_p\dot{q}\end{gathered}$$

To simplify this equation, use the robot mass matrix property [18]:

$$\dot{x}^T(\dot{M}(x) - 2C(x,\dot{x}))\dot{x} = 0 \tag{23}$$

Therefore,

$$\begin{gathered}\dot{V} = -\dot{q}^T K_v\dot{q} - \dot{q}^T D\dot{q} + \dot{q}^T(Q + rQ_G) - \dot{x}^T G(x) + \dot{q}^T K_s(\dot{L}_2 - \dot{L}_1) + \\ \left[\dot{x}^T J^T K(L_2 - L_1) - \dot{q}^T rK(L_2 - L_1) + (L_1 - L_2)^T K(\dot{L}_1 - \dot{L}_2)\right]\end{gathered} \tag{24}$$

Notice the Jacobian mapping between $\dot{L}_1$ and $\dot{x}$ and the kinematics relation between $\dot{L}_2$ and $\dot{q}$,

$$\dot{L}_1 = J\dot{x} \ , \ \dot{L}_2 = r\dot{q} \tag{25}$$

Substitute these relations into (24) and simplify:

$$\dot{V} = -\dot{q}^T(K_v + D - rK_s)\dot{q} + \dot{q}^T(Q + rQ_G) - \dot{x}^T(G(x) + J^T K_s\dot{q}) \tag{26}$$

By choosing large enough value for K_v and appropriate value for K_s, the above analysis shows that V is negative definite as long as $\dot{q}$ is not zero. This by itself is not sufficient to prove the desired stability result, since manipulator can reach a position where $\dot{q} = 0$ but $q \neq q_d$. To show that this cannot happen we show that when $\dot{q} = 0$ then $G(x) = 0$ and therefore $\dot{V} = 0$. Then LaSalle's theorem is used to show that q tend to q_d.
Suppose $\dot{q}$=0. Then $\ddot{q} = 0$ and from relations (18) and (14)

$$\begin{aligned} u &= K_p(q_d - q) + Q + rQ_G - K_s\dot{L}_1 \\ K(L_2 - L_1) &= r^{-1}u = r^{-1}[K_p(q_d - q) + Q + rQ_G - K_s\dot{L}_1]\end{aligned}$$

Therefore, the equation (13) can be rewritten in the form

$$M(x)\ddot{x} + C(x,\dot{x})\dot{x} + G(x) = r^{-1}J^T[K_p(q_d - q) + Q + rQ_G - K_s\dot{L}_1]$$

Use relations (19) and (20) and simplify:

$$M(x)\ddot{x} + C(x,\dot{x})\dot{x} = r^{-1}J^T[K_p(q_d - q) - K_s\dot{L}_1] \tag{27}$$

As we see in equation (27), $G(x) = 0$ and thus $\dot{V} = 0$. Based on LaSalle's theorem we know that the motion converges to a maximum invariant set which satisfies $\dot{V} = 0$. In this case the maximum invariant set is the responses of system (27). To show that $\dot{x}$ will tend to zero, we should prove that the above system (27) is asymptotically stable. Consider a Lyapunov function as follows:

$$W = \frac{1}{2}\dot{x}^T M(x)\dot{x} + \frac{1}{2}(q_d - q)^T K_p(q_d - q) \tag{28}$$

The time derivation of W is given by

$$\dot{W} = \dot{x}^T M(x)\ddot{x} + \frac{1}{2}\dot{x}^T \dot{M}(x)\dot{x} + (q_d - q)K_p(-\dot{q}) \tag{29}$$

By substituting equations (27) and $\dot{q} = 0$, we obtain

$$\dot{W} = \dot{x}^T[r^{-1}J^T K_p(q_d - q) - r^{-1}J^T K_s\dot{L}_1 - C(x,\dot{x})\dot{x}] + \frac{1}{2}\dot{x}^T\dot{M}(x)\dot{x} \tag{30}$$

Moreover, from relations (23) and (25) the above equation can be rewritten into

$$\dot{W} = -r^{-1}\dot{x}^T J^T K_s J\dot{x} + r^{-1}\dot{x}^T J^T K_p(q_d - q) \tag{31}$$

Define $K_s = k_s I_{n\times n}$ with $k_s > 0$, therefore matrix $J^T K_s J$ is positive defenite. By choosing appropriate values for K_s and K_p, system (27) is stable. Thus, $\dot{x}$ will tend to zero.
In this case by choosing appropriate gain matrices in proposed controller, we proved that $\dot{V}$ is zero when $\dot{q} = 0$ and as a result $\dot{x} = 0$ and hence $\ddot{q} = \ddot{x} = 0$. Therefore, from the equations of motion (13) and (14) with our proposed control (18) we must have

$$J^T K_p(q_d - q) = 0 \tag{32}$$

This equation can have both a trivial solution ($K_p(q_d - q) = 0$) and a non-trivial solution ($K_p(q_d - q) \neq 0$), because of the non-square matrix J^T. In the case of non-trivial solution, the motion stops at some point q_0 which is not equal to q_d. However, from Result 1 and Result 2 obtained from [14] based on vector closure, it is understand that the non-trivial solution cannot exist. Therefore, we conclude

$$q = q_d \tag{33}$$

as time t tends to infinity as long as the motion is within the vector closure space.

4 SIMULATIONS

A simulation study has been performed in order to verify the effectiveness of the proposed control algorithm. In the following simulation study, the results of the closed loop performance of planar cable driven manipulator examined. Our model of a planar cable robot [19] consists of a moving platform that is connected by four cables to a base platform shown in Fig. 1.

4.1 Kinematics and Jacobian

For kinematic analysis, as it is shown in Fig. 1, a fixed frame $O : xy$ is attached to the fixed base at the point O, the center of the base point circle which passes through A_i. Moreover, another moving coordinate frame $G : UV$ is located on moving platform at its center of mass G. Assume that the point A_i lies at the radial distance of R_A from point O, and the point B_i lies at the radial distance of R_B from point G in the xy plane, when the manipulator is at central location. For inverse kinematics analysis, it is assumed that the position and orientation of the moving platform $X = [x_G, y_G, \phi]^T$ is given and the problem is to find the length variable of the manipulator $l = [l_1, l_2, l_3, l_4]^T$. Let's define the instantaneous orientation angle of $B_i^{'}s$:

$$\phi_i = \phi + \theta_{B_i}$$

With some manipulation we can show that [19],

$$l_i = [(x_G - x_{A_i} + R_B cos(\phi_i))^2 + (y_G - y_{A_i} + R_B sin(\phi_i))^2]^{\frac{1}{2}}$$

For the geometry of the manipulator as illustrated in Fig. 1, the manipulator Jacobian matrix J is,

$$J = \begin{bmatrix} S_{1x} & S_{1y} & E_{1x}S_{1y} - E_{1y}S_{1x} \\ S_{2x} & S_{2y} & E_{2x}S_{2y} - E_{2y}S_{2x} \\ S_{3x} & S_{3y} & E_{3x}S_{3y} - E_{3y}S_{3x} \\ S_{4x} & S_{4y} & E_{4x}S_{4y} - E_{4y}S_{4x} \end{bmatrix} \tag{34}$$

Note that the Jacobian matrix J is a non-square 4×3 matrix, since the manipulator is redundantly actuated.

4.2 Control

The equations of motion for the end-effector can be written in the following form [20],

$$M\ddot{X} + \mathcal{G} = \mathcal{F} \ , \ X = [x_G, y_G, \phi]$$

In which, by considering flexibility in the cables,

$$\mathcal{F} = J^T K(L_2 - L_1) \ , \ L_2 = rq + L_0$$
$$I_m\ddot{q} + D\dot{q} + rK(L_2 - L_1) = \tau$$

In which,

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix} \text{ and } \mathcal{G} = \begin{bmatrix} 0 \\ -mg \\ 0 \end{bmatrix}$$

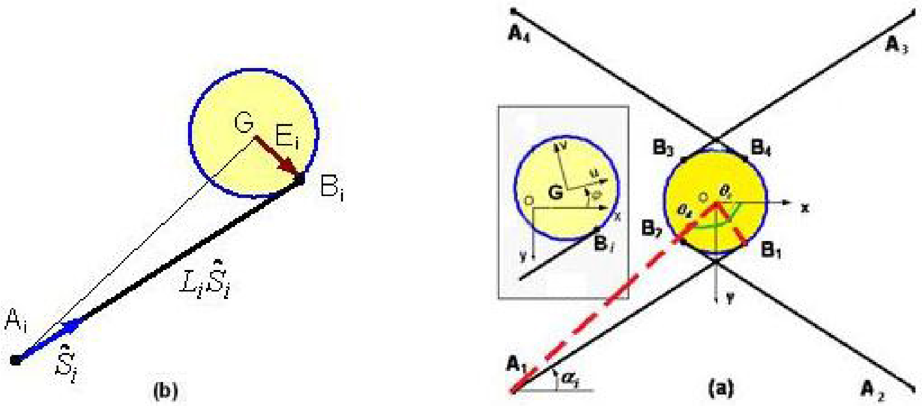

Figure 1: (a): The schematics of planar cable mechanism (b): Vector defenitions for jacobian derivation

Whose parametric values are $I_m = I_{4\times4}$, $D = 0.3\, I_{4\times4}$, $r = 1$, $K = 100\, I_{4\times4}$, $m = 2$, $I_z = 0.5$. To show the effectiveness of the proposed control algorithm suppose that we want to move the system from initial position $L_2 = [2, 1, 3, 4]$ to a fixed position $L_2 = [5, 2, 6, 8]$. The controller is based on equation (18) and consists of four parts. Controller gain matrices are chosen as $K_p = 500I$, $K_v = 100I$ and $K_s = 25I$. As illustrated in Fig. 2, the results are satisfactory and the controller achieves the desired steady state values with small errors. This simulation verifies the guaranteed stability of the proposed controller in presence of flexibility in the cables.

5 CONCLUSIONS

In this paper modeling and control of cable driven robots with cable flexibility are examined in detail. In the modeling of this kind of manipulators cables are modeled by linear axial spring, and the model of fully constrained cable driven robot is derived using Euler-Lagrange approach. Since in such robots cables must remain in tension in the whole workspace, the notion of internal force is introduced and directly used in the proposed control algorithm. The proposed control algorithm is designed in cable length space and consists of four parts. A simple PD control on the tracking error, the internal force that ensures us all of the cables are in tension, a gravity compensation term and damping term. Then, the stability of the closed-loop system is analyzed through Lyapunov second method, and it is shown that the proposed controller is capable to stabilize the system with flexible cables. Finally the performance of the proposed controller is examined through simulation.

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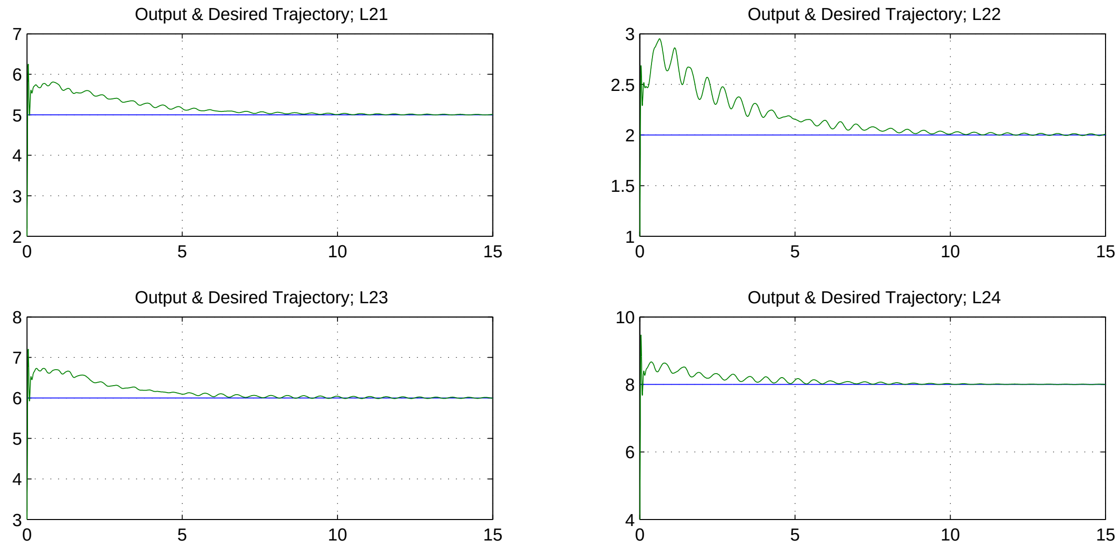

Figure 2: Plots of desired and actual position of $L_2 = rq + L_0$

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