Optimizing Fault Tolerance to Joint Jam in the Design of Parallel Robot Manipulators

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ABSTRACT

In this study, a methodology is developed to modify the design of parallel manipulators to provide fault tolerance to active joint jam (lock). The modification is based on equipping each branch in the parallel manipulator with a redundant backup active joint to which the actuation is switched in case an active joint is jammed in a branch. An optimization procedure based on linear algebra is developed to determine the optimum location and direction of each backup joint such that a dexterity-based fault tolerance measure is maximized. The manipulator is intended to be fault tolerance to jam of any of its active joints at a number of different end-effector poses.

1. INTRODUCTION

Robots used in certain applications where failure could endanger life or cause serious damage to the environment are recommended to be fault tolerant. Some of these applications are, for example, polluted areas, underwater and the outer space to which humans do not have immediate access for repair or intervention. Fault tolerant robots are capable of completing their tasks with the presence of failure.

Fault tolerance of redundant serial manipulators was studied by a number of researchers. Roberts and Maciejewski [1] studied the effect of locked joint failure on manipulability of redundant serial manipulators. The ratio of the manipulability after failure to that before failure was used as a local fault tolerance measure that indicated how close the serial manipulator came to singularity when one or more of its joints were locked at a certain manipulator pose. Paredis et al. [2] investigated the ability of redundant serial manipulators to reach certain poses after encountering joint lock.

Hassan and Notash [3] analyzed the effect of active joint jam (lock) in parallel manipulators on their velocity and static force capabilities and presented a procedure to modify the design of parallel manipulators to make them fault tolerant to active joint jam by adding a backup active joint in each branch. After encountering an active joint jam in a branch, the actuation is switched from the failed joint to the backup joint in that branch to recover the lost velocity and force capabilities. The procedure applied Lagrange Multiplier optimization method to maximize a symbolically expressed objective function, which corresponded to the ratio of manipulability after failure to that before failure at a number of specified end-effector poses [4]. Because, the Lagrange Multiplier optimization method requires obtaining partial derivatives of the symbolic objective function and subsequently solving a number of non-linear equations, finding a numerical solution becomes difficult if the symbolic objective function is highly convex, which is the case when a large number of task poses are considered in building the objective function. When designing a fault tolerant manipulator, it is important to investigate a relatively large number of poses within its workspace. This ensures that these poses sufficiently represent the workspace under consideration.

In this paper, a new method is presented which is based on applying algebraic and geometric techniques to optimize the dexterity after failure without having to use an optimization method that is based on determining function derivatives such as Lagrange multiplier optimization. The work presented focuses on the modification of parallel manipulators to make them fault tolerant by adding a backup active joint to each branch. The type, location and axis direction of the added backup joints affect the fault tolerance capability in the manipulator. In this paper, for a specific type (e.g., revolute joint) and location of the backup joint, algebraic techniques are applied to determine the optimum axis direction of that joint in each branch such that a dexterity-based fault tolerance measure is maximized. Active joints (i.e., actuated joints) are, generally speaking, more likely to jam than the passive joints (i.e., non-actuated joints). In this study, the backup active joint of each branch is designed to provide fault tolerance to active joint jam.

2. THEORETICAL BACKGROUND

2.1. Velocity of Parallel Manipulators

The velocity equation for the *i*th serial branch in a parallel manipulator is expressed in the following equation:

$$ {}^{i}\dot{\mathbf{x}} = {}^{i}J\ {}^{i}\dot{\mathbf{q}} \quad : \quad {}^{i}\dot{\mathbf{x}} \in R^{m},\ {}^{i}\dot{\mathbf{q}} \in R^{n_i},\ {}^{i}J \in R^{m\times n_i}\ (i = 1,2,\ldots,L) \qquad (1) $$

where ${}^{i}\dot{\mathbf{x}}$ is the vector composed of the components of the linear and/or angular velocity of the branch end; ${}^{i}\dot{\mathbf{q}}$ is the vector composed of velocities of the joints of the *i*th serial branch; L is the number of serial branches in a parallel manipulator; m is the dimension of the branch end motion space; n_i is the number of joints in the *i*th branch; and ${}^{i}J$ is the Jacobian matrix that maps ${}^{i}\dot{\mathbf{q}}$ to ${}^{i}\dot{\mathbf{x}}$.

Upon the jam of the *j*th joint in the *i*th serial branch of the parallel manipulator, the column corresponding to the jammed joint in the Jacobian matrix, ${}^{i}J$, is eliminated and the reduced Jacobian matrix is denoted as ${}_{-j}^{\;i}J$ and equation (1) becomes:

$$ {}^{i}\dot{\mathbf{x}} = {}_{-j}^{\;i}J \;\; {}_{-j}^{\;i}\dot{\mathbf{q}} \quad : \quad {}^{i}\dot{\mathbf{x}} \in R^{m},\; {}_{-j}^{\;i}\dot{\mathbf{q}} \in R^{n_i - 1} \text{ and } {}_{-j}^{\;i}J \in R^{m\times(n_i-1)} \quad (i = 1,2,...,L) \qquad (2) $$

where ${}_{-j}^{\;i}\dot{\mathbf{q}} = \begin{bmatrix} {}^{i}\dot{q}_1 & \dots & {}^{i}\dot{q}_{j-1} & {}^{i}\dot{q}_{j+1} & \dots & {}^{i}\dot{q}_{n_i} \end{bmatrix}^T$;

${}_{-j}^{\;i}J = \begin{bmatrix} \mathbf{c}_1({}^{i}J) & \dots & \mathbf{c}_{j-1}({}^{i}J) & \mathbf{c}_{j+1}({}^{i}J) & \dots & \mathbf{c}_{n_i}({}^{i}J) \end{bmatrix}$.

If ${}^{i}\dot{\mathbf{x}} \notin csp({}_{-j}^{\;i}J)$, the required end-effector motion ${}^{i}\dot{\mathbf{x}}$ of the *i*th branch is not permissible by the kinematics of that branch after joint jam. That is ${}^{i}\dot{\mathbf{x}} \neq \dot{\mathbf{x}}$, where $\dot{\mathbf{x}}$ is the vector composed of the components of the linear and/or angular velocity of the end-effector).

3. FAULT TOLERANCE

A parallel manipulator could be fault tolerant to active joint jam if, for example, a redundant backup active joint compensates for the reduction in the DOF and in the dimension of the task space as a result of failure. In this study, a methodology is presented to modify parallel manipulators by equipping each one of the branches with a redundant backup active joint that is kept locked and is released only in case of jam of an original active joint in that branch. By keeping these backup joints locked, the controller does not have to resolve redundancy during failure-free operation and, therefore, the controller operation could be kept simple, resulting in a less costly operation. Upon the detection and identification of jam in one of the joints in a branch, the controller initiates a backup post-failure algorithm that releases the locked backup active joint in the corresponding branch. The controller switches the actuation from the jammed joint to the backup joint and updates the kinematics of the manipulator accordingly.

When the backup active joint is released and activated to replace a jammed active joint, the dexterity of the parallel manipulator is changed. The dexterity after failure could be maximized by appropriately selecting the type, location and direction of the backup joint. This study determines the optimum location and axis direction of the backup active joint in each branch that would provide fault tolerance with maximized post-failure dexterity. In this study, it is assumed that all backup joints are revolute because revolute joints could contribute to both the translational and rotational motion of the end-effector, whereas prismatic joints could only contribute to the translational motion. By switching the actuation from the jammed joint to the backup joint in a branch, the reduced dimension of the column space of the Jacobian matrix of that particular branch is restored back to *m* and the ability of the end-effector of the parallel manipulator to arbitrarily move in the *m*-dimensional task space is recovered. In this case, another column corresponding to the backup joint is added to matrix ${}_{-j}^{\;i}J$ after switching the actuation from the failed joint *j* to the backup joint *r*. The Jacobian matrix will, then, be denoted as ${}_{-j+r}^{\;\;\;i}J$ and equation (2) could be written as:

$$ {}^{i}\dot{\mathbf{x}} = {}_{-j+r}^{\;\;\;i}J \; {}_{-j+r}^{\;\;\;i}\dot{\mathbf{q}} : \quad {}^{i}\dot{\mathbf{x}} \in R^{m},\; {}_{-j+r}^{\;\;\;i}\dot{\mathbf{q}} \in R^{n_i}, \text{ and } {}_{-j+r}^{\;\;\;i}J \in R^{m\times n_i} \quad (\text{for } i = 1,2,...,L) \qquad (3) $$

where ${}_{-j+r}^{\;\;\;i}\dot{\mathbf{q}} = \begin{bmatrix} {}_{-j}^{\;i}\dot{q}_1 & \cdots & {}_{-j}^{\;i}\dot{q}_{r-1} & {}_{-j+r}^{\;\;\;i}\dot{q}_r & {}_{-j}^{\;i}\dot{q}_r & \cdots & {}_{-j}^{\;i}\dot{q}_{n_i} \end{bmatrix}^T$;

${}_{-j+r}^{\;\;\;i}J = \begin{bmatrix} \mathbf{c}_1\left({}_{-j}^{\;i}J\right) & \cdots & \mathbf{c}_{r-1}\left({}_{-j}^{\;i}J\right) & \mathbf{c}_r\left({}_{-j+r}^{\;\;\;i}J\right) & \mathbf{c}_r\left({}_{-j}^{\;i}J\right) & \cdots & \mathbf{c}_{n_i}\left({}_{-j}^{\;i}J\right) \end{bmatrix}$; *r* is the sequence of the backup redundant joint relative to the other operating joints in the branch; ${}_{-j+r}^{\;\;\;i}\dot{\mathbf{q}}$ and ${}_{-j+r}^{\;\;\;i}J$ are the joint rate vector and Jacobian matrix, respectively, of the ith branch after switching the actuation from the jammed joint j to the backup redundant joint r; column $\mathbf{c}_r\left({}_{-j+r}^{\;\;i}J\right)$ and term ${}_{-j+r}^{\;\;i}\dot{q}_r$ correspond to the backup redundant active joint in the ith serial branch and are inserted as the rth components in ${}_{-j}^{\;i}J$ and ${}_{-j}^{\;i}\dot{\mathbf{q}}$, respectively; shifting the other components forward.

Grouping the active and passive joints separately together in the failed branch, equation (3) can be written as:

$$ {}^{i}\mathbf{x} = \left[\left({}_{-j+r}^{\;\;i}J\right)_a \quad \left({}_{-j+r}^{\;\;i}J\right)_p\right]\begin{bmatrix}\left({}_{-j+r}^{\;\;i}\dot{\mathbf{q}}\right)_a \\ \left({}_{-j+r}^{\;\;i}\dot{\mathbf{q}}\right)_p\end{bmatrix} \tag{4} $$

where $\left({}_{-j+r}^{\;\;i}J\right)_a$ and $\left({}_{-j+r}^{\;\;i}J\right)_p$ are sub-matrices of ${}_{-j+r}^{\;\;i}J$ consisting of the columns corresponding to rates of the active and passive joints of the ith branch (failed branch), respectively; and $\left({}_{-j+r}^{\;\;i}\dot{\mathbf{q}}\right)_a$ and $\left({}_{-j+r}^{\;\;i}\dot{\mathbf{q}}\right)_p$ are sub-vectors of ${}_{-j+r}^{\;\;i}\dot{\mathbf{q}}$ consisting of the rates of the active and passive joints of the failed branch, respectively.

For a 6-DOF spatial task, the end-effector velocity is the twist $\dot{\mathbf{x}} = \left[\dot{\theta}_x \quad \dot{\theta}_y \quad \dot{\theta}_z \quad \dot{x} \quad \dot{y} \quad \dot{z}\right]^T$, where $\dot{\theta}_x$, $\dot{\theta}_y$ and $\dot{\theta}_z$ are projections of the angular velocity of the end-effector onto x, y and z coordinate axes, respectively, while $\dot{x}$, $\dot{y}$ and $\dot{z}$ are projections of the linear velocity of the end-effector onto x, y and z axes, respectively. In this case, the columns of ${}_{-j+r}^{\;\;i}J$ are screws, and the column of a revolute backup $\mathbf{c}_r\left({}_{-j+r}^{\;\;i}J\right)$ in equation (3) can be written as:

$$ \mathbf{c}_r\left({}_{-j+r}^{\;\;i}J\right) = \begin{bmatrix} {}_{-j}^{\;i}\boldsymbol{a}_r \\ {}_{-j}^{\;i}\boldsymbol{a}_r \times {}^{i}\mathbf{p}_r \end{bmatrix} \tag{5} $$

where ${}_{-j}^{\;i}\boldsymbol{a}_r = \left[\left({}_{-j}^{\;i}a_r\right)_x \quad \left({}_{-j}^{\;i}a_r\right)_y \quad \left({}_{-j}^{\;i}a_r\right)_z\right]^T$ is a unit vector representing the direction of the axis of the backup redundant active joint compensating for the actuation of the failed joint (jth joint) in the ith branch; and ${}^{i}\mathbf{p}_r = \left[\left({}^{i}p_r\right)_x \quad \left({}^{i}p_r\right)_y \quad \left({}^{i}p_r\right)_z\right]^T$ represents a vector from any point on the axis of the backup redundant active joint in the ith branch to the origin of the end-effector frame. In this study, both ${}_{-j}^{\;i}\boldsymbol{a}_r$ and ${}^{i}\mathbf{p}_r$ will be represented in the global fixed frame at the base of the parallel manipulator.

The Jacobian matrix for the parallel manipulator is written as:

$$ \dot{\mathbf{x}} = {}_{-j+r}^{\;\;i}J_a \; {}_{-j+r}^{\;\;i}\dot{\mathbf{q}}_a \tag{6} $$

3.1. Fault Tolerance Measure

The absolute value of the determinant $\left|{}_{-j+r}^{\;\;i}J_a\right|$ could be used to compare the dexterity of a manipulator at different poses. Roberts and Maciejewski[1] used the ratio of the manipulability before joint failure to that after joint failure as a measure of fault tolerance to joint failure in redundant serial manipulators. In this study, the measure used for the fault tolerance of a parallel manipulator to jam of the jth joint in the

ith branch at a certain pose will be the ratio of the manipulability of the parallel manipulator after the failure and the release of the backup joint, $\left|{}_{-j+r}^{\ \ i}J_a\right|$, to that before failure, $\left|J_a\right|$ at that pose. This measure will be maximized by determining the optimum location and axis direction for the backup joints.

3.2. Optimization of Fault Tolerance Measure

As the post-failure Jacobian matrix ${}_{-j+r}^{\ \ i}J_a$ is dependent on the axis direction and location of the backup joint, its determinant $\left|{}_{-j+r}^{\ \ i}J_a\right|$ can be maximized by choosing an optimum location and axis direction for the backup joint when designing the fault tolerant manipulator. In a previous study [4], Lagrange multiplier optimization method was used to determine the optimum axis direction of backup joints, while their locations were predetermined. In the optimization method, a symbolic objective function that was based on the ratio of the manipulability after failure to that before failure was maximized. If a large number of poses are considered, the symbolic objective function becomes very large with very high convexity, which makes finding an optimum solution and numerical convergence very difficult. In this study, the optimum location and axis direction of the backup joints are determined using algebraic techniques that do not require the optimization of a symbolic objective function.

3.2.1. Fault Tolerance in a Branch with One Original Active Joint, at One Task Pose

Maximizing the determinant $\left|{}_{-j+r}^{\ \ i}J\right|$ could be achieved by choosing values for the parameters ${}_{-j}^{\ i}\boldsymbol{a}_r$ and ${}^{i}\mathbf{p}_r$ such that the following two objectives are accomplished:

(a) $\mathbf{c}_r\left({}_{-j+r}^{\ \ i}J\right)$ is as orthogonal to all other columns in ${}_{-j+r}^{\ \ i}J$ as possible (i.e., orthogonal to the columns of ${}_{-j}^{\ i}J$); and (b) the norm $\left\|\mathbf{c}_r\left({}_{-j+r}^{\ \ i}J\right)\right\|$ is maximized.

The locations of the backup joints (i.e., ${}^{i}\mathbf{p}_r$) will be pre-assigned making components of ${}_{-j}^{\ i}\boldsymbol{a}_r$ (i.e., direction cosines) the only variables. The values for these variables will be calculated through an optimization process such that the objectives (a) and (b) are accomplished. This analysis will be repeated at a number of different potential backup joint locations until the best location and axis direction for the backup joints in the branches are determined.

The orthogonal complement of the column space of ${}_{-j}^{\ i}J$ is denoted, in this study, as $csp({}_{-j}^{\ i}J)^{\perp}$ and is equal to the null space of its transpose, $nsp({}_{-j}^{\ i}J^T)$,[3] which is a one-column matrix in the case of single active joint jam. To have $\mathbf{c}_r\left({}_{-j+r}^{\ \ i}J\right)$ orthogonal to the columns of ${}_{-j}^{\ i}J$, $\mathbf{c}_r\left({}_{-j+r}^{\ \ i}J\right)$ must lie in the subspace $nsp({}_{-j}^{\ i}J^T)$. If $N({}_{-j}^{\ i}J^T)$ is a column vector forming the basis of the one-dimensional subspace $nsp({}_{-j}^{\ i}J^T)$, then, to achieve the objective of $\mathbf{c}_r\left({}_{-j+r}^{\ \ i}J\right)$ lying in $nsp({}_{-j}^{\ i}J^T)$ (i.e., objective (a)), and that of $\left\|\mathbf{c}_r\left({}_{-j+r}^{\ \ i}J\right)\right\|$ being maximized (i.e., objective (b)), one should maximize the following:

$$\max\left(\pm N\left({}_{-j}^{\ i}J^T\right)^T \mathbf{c}_r\left({}_{-j+r}^{\ \ i}J\right)\right) \tag{7}$$

The $\pm$ sign indicates that $\pm N\left({}_{-j}^{\ i}J^T\right)$ could have any of the two opposite directions. The maximum value of the product in equation (7) is equal to $\left\|\mathbf{c}_r\left({}_{-j+r}^{\ \ i}J\right)\right\| \left\|N({}_{-j}^{\ i}J^T)\right\|$ when column $\mathbf{c}_r\left({}_{-j+r}^{\ \ i}J\right)$ is orthogonal to the columns of ${}_{-j}^{\ i}J$. The objective is to determine the components of ${}_{-j}^{\ i}\boldsymbol{a}_r$ in $\mathbf{c}_r\left({}_{-j+r}^{\quad\ i}J\right)$ such that the product in equation (7) is maximized.

The unknowns in equation (7) are reduced by assuming that the locations of the backup joints, which correspond to ${}^{i}\mathbf{p}_r$, are pre-determined and that only the directions of the backup joints, which correspond to ${}_{-j}^{\ i}\boldsymbol{a}_r$, are to be calculated based on maximizing the product in equation (7). Equation (7) can be rewritten as:

$$\max\left(\pm N\left({}_{-j}^{\ i}J^T\right)^T \ {}^{i}C\begin{bmatrix}\left({}_{-j}^{\ i}a_r\right)_x\\ \left({}_{-j}^{\ i}a_r\right)_y\\ \left({}_{-j}^{\ i}a_r\right)_z\end{bmatrix}\right) \ :{}^{i}C=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 0 & \left({}^{i}p_r\right)_z & -\left({}^{i}p_r\right)_y\\ -\left({}^{i}p_r\right)_z & 0 & \left({}^{i}p_r\right)_x\\ \left({}^{i}p_r\right)_y & -\left({}^{i}p_r\right)_x & 0\end{bmatrix} \tag{8}$$

In this study, the terms $\left({}_{-j}^{\ i}a_r\right)_x$, $\left({}_{-j}^{\ i}a_r\right)_y$, and $\left({}_{-j}^{\ i}a_r\right)_z$ are expressed in the global coordinate frame. This indicates that the direction, ${}_{-j}^{\ i}\boldsymbol{a}_r$, of the backup joint in the *i*th branch changes as the link on which the backup joint is mounted moves during the motion of the manipulator from one task pose to another. If the Jacobian matrix is calculated at a task pose, denoted as *c*, that is different from that at which the direction of backup joint, ${}^{i}\boldsymbol{a}_r$, is expressed, denoted as c_o, then equation (8) is generalized to:

$$\max\left(\pm N\left({}_{-j}^{\ i}J^T\right)^T \ {}^{i}C \ {}^{i}\left(R_r\right)_{c_o,c} \ \left({}_{-j}^{\ i}\boldsymbol{a}_r\right)_c\right) \ : {}^{i}\left(R_r\right)_{c_o,c}={}^{i}\left(R_r\right)_c \, {}^{i}\left(R_r\right)_{c_o}^T \tag{9}$$

where $\left({}_{-j}^{\ i}\boldsymbol{a}_r\right)_c$ is the direction of the backup joint in the *i*th branch when the manipulator is at the initial manipulator pose c_o while $N\left({}_{-j}^{\ i}J^T\right)$ and ${}^{i}C$ are calculated at manipulator pose *c* with respect to the global frame fixed on the base; ${}^{i}\left(R_r\right)_{c_o}$ is a 3×3 rotation matrix representing the orientation of the coordinate frame of the link on which the backup joint of the *i*th branch is mounted when the manipulator is at the initial pose c_o with respect to the global frame; ${}^{i}\left(R_r\right)_c$ is a 3×3 rotation matrix representing the orientation of the coordinate frame of the link on which the backup joint of the *i*th branch is mounted when the manipulator is at pose *c* with respect to the global frame; and ${}^{i}\left(R_r\right)_{c_o,c}$ is a 3×3 rotation matrix representing the orientation of the coordinate frame of that link when the manipulator is at pose *c* with respect to the frame of the link when the manipulator is at the initial pose c_o.

If the multiplication of all the terms on the left of $\left({}_{-j}^{\ i}\boldsymbol{a}_r\right)_c$ in equation (9) is denoted as matrix ${}_{-j+r}^{\quad\ i}B_c$, then, for the case of a single active joint failure in a branch, ${}_{-j+r}^{\quad\ i}B_c$ becomes a one-row matrix. The maximum value that the term inside the brackets in equation (9) can have is when the unit vector $\left({}_{-j}^{\ i}\boldsymbol{a}_r\right)_c$ is in the same direction as that of ${}_{-j+r}^{\quad\ i}B_c$. In this case, the term inside the brackets becomes equal to $\left\|{}_{-j+r}^{\quad\ i}B_c\right\|$ as shown in the following equation:

$$ {}_{-j+r}^{\;\;i}B_c \; \left({}_{-j}^{\;i}\boldsymbol{a}_r\right)_c = \left\| {}_{-j+r}^{\;\;i}B_c \right\| \qquad : {}_{-j+r}^{\;\;i}B_c = \pm N\left({}_{-j}^{\;i}J^T\right)^T {}^iC \; {}^i\left(R_r\right)_{c_o,c} \tag{10} $$

The solution for $\left({}_{-j}^{\;i}\boldsymbol{a}_r\right)_c$ in equation (10) is:

$$ \left({}_{-j}^{\;i}\boldsymbol{a}_r\right)_c = \frac{{}_{-j+r}^{\;\;i}B}{\left\| {}_{-j+r}^{\;\;i}B \right\|} \qquad : {}_{-j+r}^{\;\;i}B_c = \pm N\left({}_{-j}^{\;i}J^T\right)^T {}^iC \; {}^i\left(R_r\right)_{c_o,c} \tag{11} $$

This solution maximizes the product inside the brackets in equations (7)-(9) and constitutes an optimum direction of the backup joint that maximizes the post-failure manipulability $\left| {}_{-j+r}^{\;\;i}J \right|$ at manipulator pose c (expressed in the global coordinate frame when the link on which that joint is mounted is at manipulator pose c_o). The direction $\left({}_{-j}^{\;i}\boldsymbol{a}_r\right)_c$ calculated from equation (11) could have a positive or a negative sign.

After determining the optimum direction $\left({}_{-j}^{\;i}\boldsymbol{a}_r\right)_c$, the ratio of the manipulability after failure, which is a function of $\left({}_{-j}^{\;i}\boldsymbol{a}_r\right)_c$, to that before failure in case of the jth joint jam in the ith branch at manipulator pose c is calculated as:

$$ {}^if\left({}_{-j}^{\;i}\boldsymbol{a}_r\right)_c = abs\left(\frac{\left| {}_{-j+r}^{\;\;i}\boldsymbol{J}_a \right|}{\left| \boldsymbol{J}_a \right|} \right) \tag{12} $$

The ratio in equation (12) is used as a measure of fault tolerance for a single active joint failure at a single task pose and vector $\left({}_{-j}^{\;i}\boldsymbol{a}_r\right)_c$ calculated from equation (11) maximizes the measure. If the branch has more than one active joint, the backup joint direction $\left({}_{-j}^{\;i}\boldsymbol{a}_r\right)_c$ in that branch should be calculated such that the backup joint provides fault tolerance to failure to any of the active joints. In addition, vector $\left({}_{-j}^{\;i}\boldsymbol{a}_r\right)_c$, calculated from equation (11), is not optimum if failure occurs at a manipulator pose other than c. In the situation where the backup joint is expected to provide fault tolerance to failure at all poses within a workspace, the backup joint direction $\left({}_{-j}^{\;i}\boldsymbol{a}_r\right)_c$ should be calculated such that the joint is capable of providing fault tolerance at a number of task poses in that workspace.

3.2.2. Fault Tolerance in a Branch with Multiple Active Joints, and Various Task Poses

If the total number of task poses, at which the manipulator is required to be fault tolerant, is c_t and the ith branch has n_{ia} active joints, then there will be ($c_t \times n_{ia}$) different backup joint directions, $\left({}_{-j}^{\;i}\boldsymbol{a}_r\right)_c$, calculated from equation (11). Each one of these directions will be optimum for one of the failure cases in which one of the active joints (j=1,…, n_{ia}) in the ith branch fails while the manipulator is at one of the task poses (c=1,…, c_t) within the workspace. Since the backup joint in a branch provides fault tolerance to these failure cases one at a time, one needs to determine a fault tolerance measure that combines all these failure cases such that a high value for this measure indicates improved fault tolerance for all failure cases. In this section, this measure is formulated and a procedure is presented to maximize it.

To simplify the notation $\left({}_{-j}^{\;i}\boldsymbol{a}_r\right)_c$, it will be replaced by ${}^i\mathbf{s}_k$ where index number k=1,…,$c_t \times n_{ia}$. Index k represents a failure case (i.e., failure of one of the n_{ia} active joints at one of the c_t task poses) for the ith branch. Vectors ${}^i\mathbf{s}_k$ (k=1,…,$c_t \times n_{ia}$) are grouped in matrix ${}^iS$ as:

$$ {}^{i}S = \pm\left[{}^{i}\mathbf{s}_1 \quad \cdots \quad {}^{i}\mathbf{s}_k \quad \cdots \quad {}^{i}\mathbf{s}_{n_{ia}\times c_t}\right] \tag{13} $$

The vectors of matrix ${}^{i}S$ are the optimum directions calculated from equation (11) for each one of the (c_t×n_{ia}) failure cases. The manipulability ratio calculated from equation (12) for each optimum direction in ${}^{i}S$ are listed as:

$$ {}^{i}\mathbf{f} = \left[{}^{i}f_1 \quad \cdots \quad {}^{i}f_k \quad \cdots \quad {}^{i}f_{n_{ia}\times c_t}\right]^T \qquad : {}^{i}f_k = {}^{i}f\left({}^{i}\mathbf{s}_k\right) \tag{14} $$

The term ${}^{i}f\left({}^{i}\mathbf{s}_k\right)$ is basically just an alternative notation to ${}^{i}f\left({}_{-j}^{\;i}\boldsymbol{a}_r\right)_c$. The optimum direction of the backup joint in the ith branch for all failure cases k=1, .., n_{ia}×c_t, will be denoted as ${}^{i}\mathbf{s}_{opt}$ and will be calculated from the vectors in ${}^{i}S$. This optimum direction is a function of all the vectors ${}^{i}\mathbf{s}_k$ (k=1,.., n_{ia}×c_t). Before these vectors are used to calculate ${}^{i}\mathbf{s}_{opt}$, one needs to assign a weighting factor to each one of them. A weighting factor associated with ${}^{i}\mathbf{s}_k$ represents the capability of a backup joint with an axis direction ${}^{i}\mathbf{s}_k$ to provide tolerance not only to failure case k, for which it is determined as optimum, but also to all failure cases other than k as well. To calculate the weighting factors associated with each vector ${}^{i}\mathbf{s}_k$, the ratio in equation (12) are calculated for each ${}^{i}\mathbf{s}_k$ (k=1,.., n_{ia}×c_t) in each one of the n_{ia}×c_t failure cases and the results are grouped in a (n_{ia}×c_t)×(n_{ia}×c_t) matrix ${}^{i}G$ written as:

$$ {}^{i}G = \begin{bmatrix} {}^{i}f_{1,1} & \cdots & {}^{i}f_{1,k} & \cdots & {}^{i}f_{1,(n_{ia}\times c_t)} \\ {}^{i}f_{g,1} & & {}^{i}f_{g,k} & & {}^{i}f_{g,(n_{ia}\times c_t)} \\ {}^{i}f_{(n_{ia}\times c_t),1} & & {}^{i}f_{(n_{ia}\times c_t),k} & & {}^{i}f_{(n_{ia}\times c_t),(n_{ia}\times c_t)} \end{bmatrix} \tag{15} $$

where ${}^{i}f_{g,k}$ is the manipulability ratio corresponding to failure case g calculated from equation (12) using vector ${}^{i}\mathbf{s}_k$, which is considered optimum for failure case k. The terms in ${}^{i}\mathbf{f}$ are for the cases when g=k and these terms constitute the diagonal entries in ${}^{i}G$.

The weighting factors are calculated by multiplying the components in each column of matrix ${}^{i}G$ together and are listed as:

$$ {}^{i}\mathbf{w} = \left[{}^{i}w_1 \quad \cdots \quad {}^{i}w_k \quad \cdots \quad {}^{i}w_{n_{ia}\times c_t}\right]^T \tag{16} $$

where ${}^{i}w_k = \prod_{g=1}^{n_{ia}\times c_t} {}^{i}f_{g,k}$.

If one term in ${}^{i}G$ is relatively very low, it indicates relatively poor fault tolerance provided by a backup joint with the corresponding direction in the corresponding failure case. This low value results in low value for the weighting factor ${}^{i}w_k$ associated with that axis direction. The higher the value of ${}^{i}w_k$, the more capable a backup joint with axis direction ${}^{i}\mathbf{s}_k$ is in providing fault tolerance to active joint jam for any of the failure cases k=1,.., $n_{ia}\times c_t$ in the ith branch. Multiplying each vector ${}^{i}\mathbf{s}_k$ in matrix ${}^{i}S$ by its weighting factor results in a matrix ${}^{i}S_w={}^{i}S\ {}^{i}\mathbf{w}$ as shown in equation (17):

$$ {}^{i}S_w = \pm\left[{}^{i}w_1\,{}^{i}\mathbf{s}_1 \quad \cdots \quad {}^{i}w_k\,{}^{i}\mathbf{s}_k \quad \cdots \quad {}^{i}w_{n_{ia}\times c_t}\,{}^{i}\mathbf{s}_{n_{ia}\times c_t}\right] \tag{17} $$

The vectors in ${}^{i}S_w$ belonging to R^3-space are sketched in Figure 1. The optimum direction, ${}^{i}\mathbf{s}_{opt}$, will be biased (or close) to the direction of the vectors with the highest weighting factors in Figure 1.

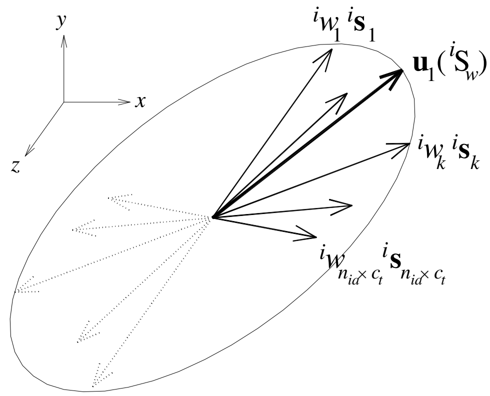

Figure 1. The optimum weighted vectors ${}^{i}\mathbf{s}_k$ directed toward the vectors with the higher weights.

Because the vectors in ${}^{i}S_w$ are determined based on the null space analysis in equation (7), each vector can have one of two opposite senses. Before determining ${}^{i}\mathbf{s}_{opt}$ from these vectors, one of the two senses has to be eliminated. The sense to eliminate is chosen based on the objective that these vectors should have senses in the R^3-space such that the remaining vectors are clustered towards the vectors with the highest weights. As each one of these vectors is a unit vector scaled by its weighting factor, the sense associated with the highest weighting factor in the R^3-space can be determined approximately to be along the singular vector associated with the largest singular value calculated from the singular value decomposition of matrix ${}^{i}S_w$. The largest singular vector is denoted as $\mathbf{u}_1\left({}^{i}S_w\right)$. To direct all the vectors in ${}^{i}S_w$ to approximately face the direction of the singular vector $\mathbf{u}_1\left({}^{i}S_w\right)$, all vectors making negative scalar products with $\mathbf{u}_1\left({}^{i}S_w\right)$ will be eliminated. This technique realigns the vectors in ${}^{i}S_w$ to be clustered along the direction of the vectors with larger weights.

The optimum vector, ${}^{i}\mathbf{s}_{opt}$, is determined to be along the direction of the resultant of all the realigned vectors in ${}^{i}S$ as shown in the following equation:

$$ {}^{i}\mathbf{s}_{opt} = \frac{\sum_{k=1}^{n_{ia}\times c_t} {}^{i}\mathbf{s}_k}{\left\| \sum_{k=1}^{n_{ia}\times c_t} {}^{i}\mathbf{s}_k \right\|} \tag{18} $$

The optimum direction of the backup joint, ${}^{i}\mathbf{s}_{opt}$, is calculated separately for each branch as each backup joint provides fault tolerance to active joint failure within its branch only. The ratio of the manipulability after failure to that before failure when the backup joint axis direction is ${}^{i}\mathbf{s}_{opt}$ are calculated for each failure case for the active joints in the ith branch and are listed as:

$$ {}^{1}\mathbf{f}_{opt} = \left[{}^{i}f_{1,opt} \quad \cdots \quad {}^{i}f_{k,opt} \quad \cdots \quad {}^{i}f_{n_{ia}\times c_t,opt} \right]^T \tag{19} $$

where ${}^{i}f_{k,opt}$ is fault tolerant measure corresponding to failure case k for the ith branch calculated from equation (12) using the optimum vector ${}^{i}\mathbf{s}_{opt}$.

To evaluate the direction ${}^{i}\mathbf{s}_{opt}$ relative to ${}^{i}\mathbf{s}_k$ (k=1,.., $n_{ia}\times c_t$), a weighting factor, denoted as ${}^{i}w_{opt}$ and associated with ${}^{i}\mathbf{s}_{opt}$, is calculated by multiplying all the values in ${}^{i}\mathbf{f}_{opt}$ together as shown in the following equation:

$$ {}^{i}w_{opt} = \prod_{k=1}^{n_{ia}\times c_t} {}^{i}f_{k,opt} \tag{20} $$

If the value of ${}^{i}w_{opt}$ is greater any of the weight values in equation (16), then it indicates that a backup joint with direction ${}^{i}\mathbf{s}_{opt}$ provides more improved fault tolerance for all the different failure cases than a backup joint with any of the directions in ${}^{i}S$. Each direction ${}^{i}\mathbf{s}_k$ (k = 1, .., $n_{ia}\times c_t$) calculated from equation (11), is optimum for its corresponding failure case only, but ${}^{i}\mathbf{s}_{opt}$ is optimum for all the different failure cases in the branch. Adding a backup revolute joint with direction ${}^{i}\mathbf{s}_{opt}$ at the pre-assigned location in the ith branch provides optimum fault tolerance for failure of any of the n_{ia} active joints in that branch at any of the c_t manipulator poses. The optimum direction ${}^{i}\mathbf{s}_{opt}$ is determined when the backup joint is at the pre-assigned location. A different optimum direction ${}^{i}\mathbf{s}_{opt}$ and a different ${}^{i}w_{opt}$ are determined when the backup joint is pre-assigned at a different potential location on the branch. The location that results in a larger value of ${}^{i}w_{opt}$ is considered to be better. This process could be repeated for other potential backup joint locations in each branch separately until the optimum location and direction of the backup joints in all the branches are identified.

4. EXAMPLE

The example parallel manipulator of Ref. [5], which is shown in Figure 2, consists of a fixed base platform and three identical branches carrying a mobile platform (end-effector).

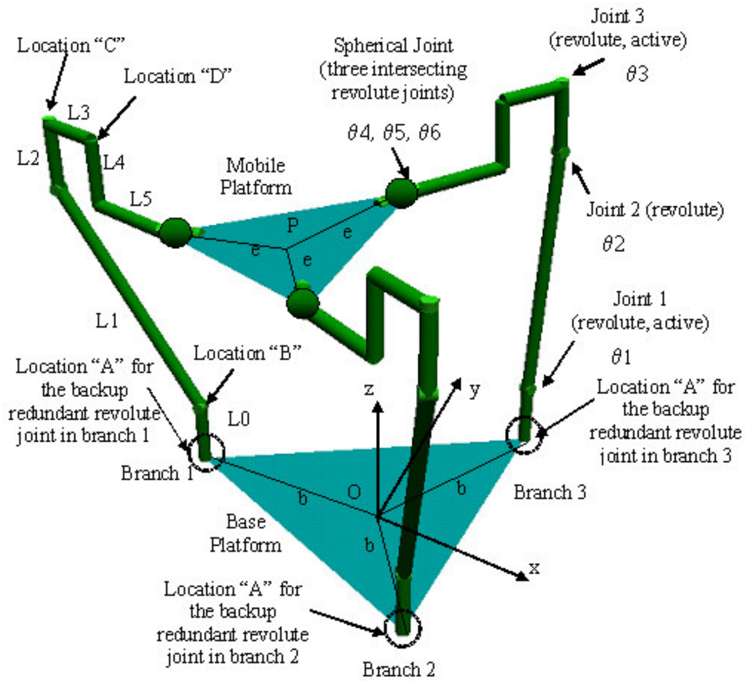

Figure 2. Layout of the example 6-DOF spatial parallel manipulator.

The base and mobile platforms form two equilateral triangles with centre points O and P, respectively. The three branches are joined to the base platform at the corners. Each branch consists of six revolute joints, where the three distal joints form a spherical joint group. The axes of joints 1 and 2 in each branch are parallel together and to the base platform while the axis of joint 3 is perpendicular to that of joint 2. In each branch, the first and the third joints are active. The values of the lengths shown in Figure 2 are: e=0.2 m, b=0.3 m, L0=0.1 m, L1=0.4 m, L2=L4=0.1 m, L3=0.1 m, and L5=0.15 m. The origin of the global x-y-z coordinate system is located at point O on the base platform. The parallel manipulator of Figure 2 is modified by adding a backup redundant joint in branch 1 to provide fault tolerance against jam of any of the two active joints (i.e., joints 1 and 3) in that branch. The methodology explained in the previous section is employed to find the axis direction of the redundant backup joint in each branch to provide optimized fault tolerance against active joint jam in that branch when the parallel manipulator is at the pose in which the orientation of the mobile platform (in degrees) and the position of point P on the mobile platform (in meters) are [θ_x , θ_y , θ_z , p_x , p_y , p_z]= [0, 0, 0, 0, 0, 0.416]. The analysis is conducted when the backup joint is at location A at the base of the branch (shown in Figure 2) on branch 1. With the backup joint being at location A, the position vector $\left({}^{1}\mathbf{p}_1\right)_1$ which represents the position of the reference frame origin of the backup joint in branch 1 with respect to point P on the mobile platform, expressed in the global frame at the base, is calculated as $\left({}^{1}\mathbf{p}_1\right)_1 = [0.300 \quad 0 \quad 0.416]^T$ m. Calculating the Jacobian matrices for branches 1, 2 and 3 (i.e., ${}^{1}J$, ${}^{2}J$ and ${}^{3}J$), inverting these and grouping the rows corresponding to the active joints, the Jacobian matrix J_a for the healthy parallel manipulator can be calculated.

In the cases of failure of joint 1 and joint 3, the vectors orthogonal to the column spaces of the two Jacobian matrices ${}_{-1}^{1}J$ and ${}_{-3}^{1}J$ are $N\left({}_{-1}^{1}J^T\right)$ = ± [0, 0.044, 0, –0.973, 0, 0.225]^T and $N\left({}_{-3}^{1}J^T\right)$ = ±

$[0.001, 0, 0.196, 0, -0.981, 0]^T$, respectively. Each one of these vectors is orthogonal to all the columns in the branch Jacobian matrix that correspond to the healthy joints in branch 1 at pose 1 and represents a constraining wrench applied on end-effector motion due to joint failure. The rotation matrix ${}^{1}(R_1)_{1,1}$ corresponding to ${}^{i}(R_r)_{c_o,c}$ in equation (9) is an identity matrix in both failure cases (i.e., failure of both active joints). The optimum axis directions of the backup joint in branch 1 at pose 1 in the cases of failure of joint 1 and joint 3 can be calculated from equation (11) as: $\left({}_{-1}^{1}\boldsymbol{a}_1\right)_1 = \pm[0, -1, 0]^T$ and $\left({}_{-3}^{1}\boldsymbol{a}_1\right)_1 = \pm[0.972, 0, -0.234]^T$, respectively. If these optimum directions are substituted in matrices ${}_{-1+1}^{1}\boldsymbol{J}$ and ${}_{-3+1}^{1}\boldsymbol{J}$, the ratio of manipulability after failure to that before failure calculated from equation (12) in each of the two failure cases in branch 1, are 1.294 and 1.716, respectively. These values indicate an increase in the manipulability after failure from its value before failure if the actuation is switched from any of the two active joints (j=1 or j=3) to the backup joint with an optimum direction $\pm[0, -1, 0]^T$ or $\pm[0.972, 0, -0.234]^T$, respectively. The optimum backup joint direction for branch 1 for both failure cases is determined from equation (18) as ${}^{1}\mathbf{s}_{opt}$ =[–0.688, 0.707, 0.165] and the fault tolerance measures in equation (19) are determined as ${}^{1}f_{1,opt}$=0.915, for failure of joint 1, and ${}^{1}f_{2,opt}$=1.214, for failure of joint 2. These values prove that the backup joint at location A with a direction ${}^{1}\mathbf{s}_{opt}$ is capable of restoring the manipulability of the manipulator after encountering jam of any of the two active joints in branch 1 at the given pose. This procedure could be repeated for branches 2 and 3 to determine the optimum axis direction of the backup joints for these branches.

5. CONCLUSION

In this study, the concept of modifying the design of parallel manipulators by including a backup joint in each branch to provide fault tolerance in the event an active joint in the branch jams is presented. The location and direction of the backup joint in each branch are optimized based on maximizing the manipulability of the parallel manipulator after switching the actuation from the failed active joint to the backup joint in that branch. A methodology based on linear algebra was proposed to determine the optimum locations and directions of backup joints that are capable of delivering fault tolerance to all the active joint failure cases. This methodology is efficient in optimizing fault tolerance at a large number of failure cases because it is based on applying algebraic operations on vectors and matrices and is not based on differentiating symbolic objective functions. The proposed methodology combines numerical results from all failure cases in matrices, on which algebraic operations are performed to extract an optimum solution for the backup joints directions. The proposed technique becomes very attractive when the manipulator is intended to be fault tolerant in a workspace represented by a large number of poses.

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