SYNTHESIS OF HYPOCYCLOIDAL GEARS

R. Dolchinkov, V. Galabov*, N. Nikolov*, and V.N. Latinovic**

1. Abstract

The article presents the results of analytical study of hypocycloidal gearsets, generated by a known compound pinion consisting of a hub and cylindrical teeth that meshes with hypocycloidal gear with internal teeth; the number of teeth of the pinion being for one smaller than the number of teeth the hypocycloidal gear. Two gearsets are synthesized in which the compound pinion has been replaced by an integral pinion with tooth profile generated by an external and internal envelope resulting from successive positions of tooth profile of the hypocycloidal gear according to the conjugate action of the two in the plane perpendicular to the axes of centrodes.

2. Introduction

In spite of undisputable advantages of the involute gearsets due to their insensitivity to center distance mounting error, constant pressure angle and easy machining, cycloidal gears are becoming more and more attractive to researchers in exploring their kinematic and load transmission characteristics. The cycloidal gears are characterized by a large gear ratio if they are designed in a planetary arrangement. This is due to the fact that the number of teeth of pinion is for one smaller than the number of teeth of gear. The other advantage is in their higher load rating and efficiency for equal size of the gearset. Because of these advantages the overall share in gear applications in practice increases every year in favor of the cycloidal gears [1]. This claim can be guessed based on fact that a convex and a concave surface are in contact during the meshing; yet it has to be proven.

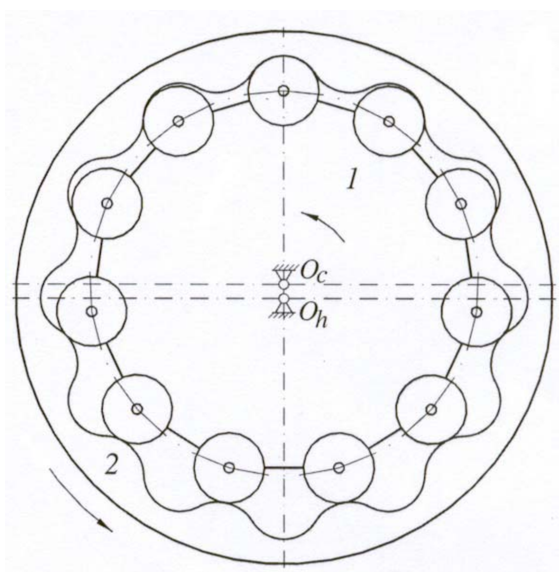

Fig. 1 Compound Pinion with Cylindrical Teeth and Epicycloidal Gear with m=3.5 mm, N_1 = 11, N_2= 12, x =0.2 and $r_c^* = 1$

A well known hypocycloidal gearset with internal gear is one with compound pinion [2], [3]. The tooth profile of the internal gear is an equidistant curve of the shortened epicycloid, and the teeth of the pinion are cylinders; the number of cylinders being for one smaller that number of epicycloidal teeth. Fig. 1 reveals this type of gearset. At any instant a half of the meshing tooth pairs is in contact and transmits motion to the driven gear.

Dolchinkov [4] has synthesized and studied a new type of hypocycloidal gear , generated by a compound pinion with cylindrical teeth. In this case the compound pinion is replaced by an integral pinion with external tooth profile generated as an inner envelope curve of the hypocycloidal gear according to the conjugate action resulting from pure rolling of the centrodes. The hypocycloidal gear (with internal teeth) and the epicycloidal pinion constitute a new gearset. Compared to the gearset with the compound pinion and cylindrical teeth this gearset it is characterized by easy manufacturing and higher reliability and load rating.

In order to make a complete classification of hypocycloidal gearsets it is necessary to consider a new type of gearset, generated by a pinion with epicycloidal tooth profile. The tooth profile of the hypocycloidal gear with internal teeth is used instead of the compound pinion with cylindrical teeth. It is generated as an outer envelope curve of tooth profile of the hypocycloidal gear in the plane perprndicular to axes of the centrods that roll with no slippage on each other. In this articles the authors are presenting synthesis of a class of hypocycloidal gearsets. The synthesis is accomplished based on either the fundamental low of gearing [5] or on the conjugate action and the theory of envelopes [6].

3. Synthesis of Hypocycloidal Gearsets

The central curve of the cylinders of a compound pinion 1 in the plane of the hypocycloidal gear 2 is a shortened epicycloid described by a parametric equations (Fig. 2):

$$\xi_c = \frac{m}{2}[(N_2+1)\sin\varphi - (1-x)\sin(N_2+1)\varphi], \quad \eta_c = \frac{m}{2}[(N_2+1)\cos\varphi - (1-x)\cos(N_2+1)\varphi] \tag{1}$$

where m, N_1 and x are a module, number of teeth and modification factor of the epicycloidal pinion respectively, and angle φ assumes value within range $[0, \frac{\pi}{N_2}]$.

The equations of the tooth-profile of the hypocycloidal gear that turns to be an equidistant curve of the shortened epicycloid (1), are:

$$\xi = \xi_c + r_c\frac{(1-x)\sin(N_2-1)\varphi - \sin\varphi}{\sqrt{1-2(1-x)\cos N_2\varphi + (1-x)^2}}, \quad \eta = \eta_c + r_c\frac{-(1-x)\cos(N_2-1)\varphi - \cos\varphi}{\sqrt{1-2(1-x)\cos N_2\varphi + (1-x)^2}} \tag{2}$$

where $r_c = m r_c^*$ is the radius of generating circle (equal to radius of the cylinder), r_c^* is termed a coefficient of generating cirle, and N_e is number of teeth of epicycloidal gear.

To replace the compound pinion with cylindrical teeth by an integral pinion its tooth-profile must be an envelope curve of the succesive positions of the tooth-profile of the epicycloidal gear in the plane perpendicular to axes of the centrodes that roll with no slippage on each other. The equations of a curve from the family of curves can be obtained by a kinematic inversion of the gear motion relative to the pinion motion. Then, commonly, one can assume that the fixed frame

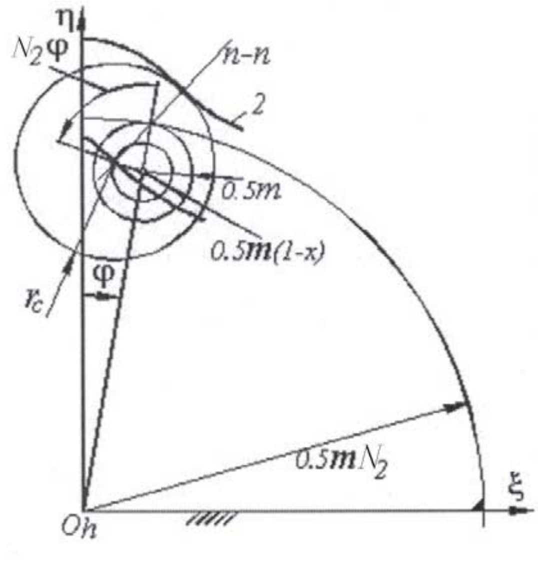

Fig. 2 Generating Tooth Profile of Epiocycloidal Gear

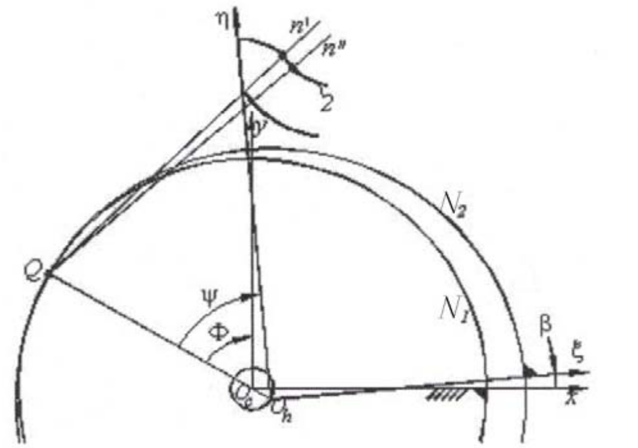

Fig. 3 Generating Two Families of Curves in Plane of Centrods

is attached to the pinion $O_c xy$ (does not move) (Fig.3). The hypocycloidal gear is fixed to frame $O_h\xi\eta$ and produces an angular displacement in plain $O_c xy$. Equations of a family of curves, corresponding to the succesive positions of the shortened epicycloid (1) and its equidistant tooth profile of the hypocycloidal gear are obtained by the following coordinate transformations:

$$x_c = \xi_c \cos\beta - \eta_c \sin\beta - a_\omega \sin\phi, \quad y_c = \xi_c \sin\beta + \eta_c \cos\beta - a_\omega \cos\phi \tag{3}$$

$$x = \xi\cos\beta - \eta\sin\beta - a_\omega \sin\phi, \quad y = \xi\sin\beta + \eta\cos\beta - a_\omega \cos\phi \tag{4}$$

where $\phi \in [0,2\pi]$ is an angle that defines the position of the centre O_h of the hypocycloidal gear *1* (Fig.2), $\beta=\phi/N_1=\psi/N_2$ is an angle of rotation of the hypocycloidal gear relative to the pinion wheel, where ψ defines the position of instantaneous center Q in the coordinate system of the hypocycloidal gear; $N_2 = N_1 + 1$; N_1 being the number of teeth (cylinders) of the compound gear, and $a_\omega = m(1-x)/2$ the center distance.

By substituting equations (1) and (2) into (3) and (4) the following is obtained:

$$x_c = \frac{m}{2}[N_1 \sin(\varphi + \frac{\psi}{N_2}) - (1-x)\sin(N_1\varphi - \frac{\psi}{N_1}) - \lambda\sin(\frac{N_2}{N_1}\psi)] \tag{5a}$$

$$y_c = \frac{m}{2}[N_1 \cos(\varphi + \frac{\psi}{N_2}) + (1-x)\cos(N_1\varphi - \frac{\psi}{N_1}) - \lambda\cos(\frac{N_2}{N_1}\psi)] \tag{5b}$$

$$x = \frac{m}{2}[N_1 \sin(\varphi + \frac{\psi}{N_1}) - \lambda \sin(N_1\varphi - \frac{\psi}{N_1}) - \lambda \sin(\frac{N_2}{N_1}\psi) + 2r_c^* \frac{\lambda \cos(N_p\varphi - \frac{\psi}{N_1}) - \cos(\varphi - \frac{\psi}{N_1})}{\sqrt{1 - 2\lambda \cos N_2\varphi + \lambda^2}}] \quad (6a)$$

$$y = \frac{m}{2}[N_1 \cos(\varphi + \frac{\psi}{N_1}) + \lambda \cos(N_1\varphi - \frac{\psi}{N_1}) - \lambda \cos(\frac{N_2}{N_1}\psi) + 2r_c^* \frac{\lambda \cos(N_p\varphi - \frac{\psi}{N_1}) - \cos(\varphi - \frac{\psi}{N_1})}{\sqrt{1 - 2\lambda \cos N_2\varphi + \lambda^2}}] \quad (6b)$$

where λ designates factor *(1-x)*.

The envelopes of the family of curves, corresponding to the succesive positions of tooth-profile of the epicycloidal gear in the plane perpendicular to the axes of the centrods are defined by system:

$$y - y(\varphi(x), \phi_{1,0}) = 0 \quad (7a)$$

$$\frac{dy_c}{d\phi_{0,1}} = 0, \text{ where } \phi_{1,0} = \phi \quad (7b)$$

Equation (7a) is equivalent to equations $y - y(\varphi, \phi) = 0$, $x - x(\varphi, \phi) = 0$ and. Since the shortened epicycloid (1) and the tooth-profile of the hypocycloidal gear (2) are equidistant curves the slopes are the same, and hence $dy/d\phi \equiv dy_c/d\phi$. Equation (7a) can be rewritten as follows:

$$\frac{dy_c}{d\phi_{0,1}} = (y_c)'_\varphi \frac{d\varphi(x_c)}{d\phi_{0,1}} + (y_c)'_{\phi_{1,0}} \frac{d\phi_{0,1}}{d\phi_{0,1}} = -(y_c)'_\varphi \frac{d\varphi(x_c)}{d\phi_{1,0}} + (y_c)'_{\phi_{1,0}} = -(y_c)'_\varphi (\varphi)'_{x_c} \frac{dx_c}{d\phi_{1,0}} + (y_c)'_{\phi_{1,0}} =$$

$$= -\frac{(y_c)'_\varphi}{(x_c)'_\varphi}(x_c)'_{\phi_{1,0}} + (y_c)'_{\phi_{1,0}} = 0 \quad .$$

Finally, the following system is obtained:

$$x - x(\varphi, \phi_{1,0}) \equiv x - x(\varphi, \psi) = 0; \quad (8a)$$

$$y - y(\varphi, \phi_{1,0}) \equiv y - y(\varphi, \psi) = 0 \quad (8b)$$

$$-\frac{(y_c)'_\varphi}{(x_c)'_\varphi} + \frac{(y_c)'_{\phi_{1,0}}}{(x_c)'_{\phi_{1,0}}} \equiv \frac{(y_c)'_\varphi}{(x_c)'_\varphi} + \frac{(y_c)'_\psi}{(x_c)'_\psi} = 0 \quad (8c)$$

After differentiating equations (5) with respect to two parametres φ and ψ and substituting into equation (8b) the following result is obtained:

$$\frac{\frac{m}{2}N_1[\sin(\varphi + \frac{\psi}{N_1}) + \lambda \sin(N_1\varphi - \frac{\psi}{N_1})]}{\frac{m}{2}N_1[\cos(\varphi + \frac{\psi}{N_1}) - \lambda \cos(N_1\varphi - \frac{\psi}{N_1})]} - \frac{\frac{m}{2N_1}[N_1 \sin(\varphi + \frac{\psi}{N_1}) - \lambda \sin(N_1\varphi + \frac{\psi}{N_1}) - \lambda N_2 \sin(\frac{N_2}{N_1}\varphi)]}{\frac{m}{2N_1}[N_1 \cos(\varphi + \frac{\psi}{N_1}) + \lambda \cos(N_1\varphi - \frac{\psi}{N_1}) - \lambda N_2 \cos(\frac{N_2}{N_1}\psi)} = 0$$

The above equation can be reduced to the following simplified form:

$$\lambda \sin[(\psi-\varphi)+N_1\varphi]-\sin(\psi-\varphi)-\sin(N_2\varphi)=0 \qquad (9)$$

which reduces to a quadric equation:

$$(\lambda^2-2\lambda\cos N_2\varphi+1)\cos^2(\psi-\varphi)+2\lambda\sin^2 N_2\varphi\cos(\psi-\varphi)+[2\lambda-(1+\lambda^2)\cos N_2\varphi]\cos N_2\varphi=0 \qquad (10)$$

with the two roots:

$$\psi_{3,4}(\varphi)=\varphi+\cos^{-1}\frac{-\lambda\sin^2 N_2\varphi\mp\sqrt{\lambda^2\sin^4 N_1\varphi-[\lambda^2-2\lambda\cos N_2\varphi+1][2\lambda-(1+\lambda^2)\cos N_2\varphi]\cos N_2\varphi}}{\lambda^2-2\lambda\cos N_2\varphi+1} \qquad (11)$$

The relationship (11) between parametres ψ and φ is substituted into the first two equations of system (8) and then into equations (6) to yield the outer and inner envelopes of successive positions of tooth profile of the hypocycloidal gear as a function of the parameter φ. The envelopes of the equidistant curve of the tooth profile of the epicycloidal gear for the shortened hypocycloid are obtained by substituting values (11) into equations (5). Fig.4 shows functions $\psi_3(\varphi)$ and $\psi 4(\varphi)$ and Fig.5 shows the tooth profile of the hypocycloidal gear 2 , cylinder 1 of the compound pinion and the tooth profiles of gears *3* and *4* A_3BC_3 and A_4BC_4, representing outer and inner envelope of the tooth profile of the epicycloidal gear in the plane perpendicular to axes of centrods for N_1=11, $N_2 \equiv N_3 \equiv N_4$=12 , x = 0.2 and $r_c^*=1$.

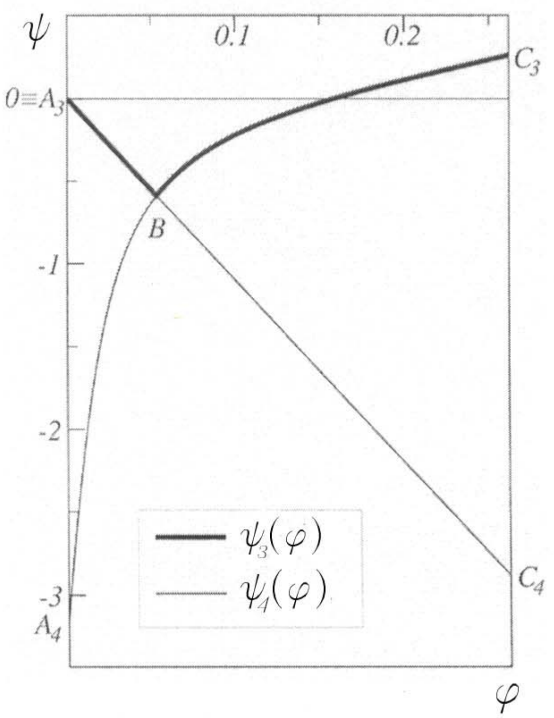

Fig. 4 Relationship Between ψ and φ for N_1 = 11, N_2 = 12 and x = 0.2

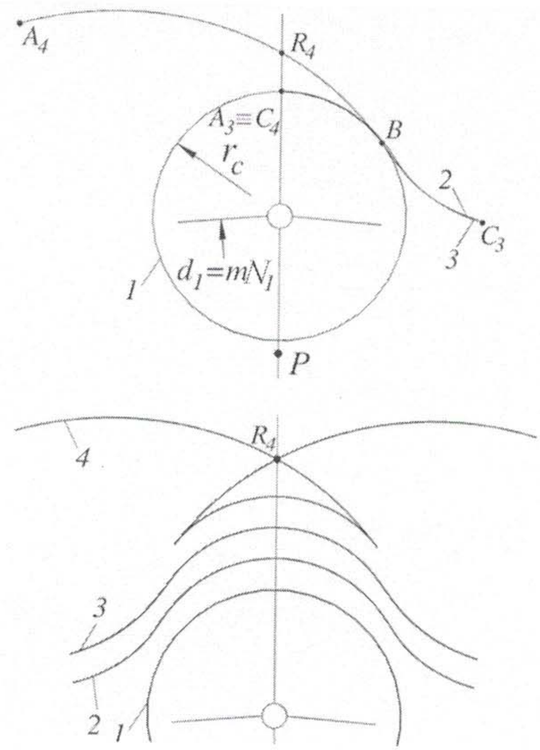

Fig. 5 Two Envelopes for N_1 = 11, N_2 = N_3 = N_4 =12 , x = 0.2 and $r_c^*=1$

Besides the approach, based on the theory of envelopes demonstrated above, it is also possible to use the approach reported in [4] and [7], based on the well known principle of kinematics the fundamental low of gearing for higher pairs in contact. According to this low the common normal at the contact point of the two profiles always passes through the instantaneous center on the line of centers and it is designated by P. Since the slope of the normal is –1/slope of curve at a point, it can be written in reference system of hypocycloidal gear $O_e\xi\eta$:

$$\frac{\eta_c - \eta_P}{\xi_c - \xi_P} = -\frac{1}{d\eta_c / d\xi_c} = -\frac{d\xi_c / d\varphi}{d\eta_c / d\varphi} \tag{12}$$

Derivatives $d\xi_c/d\varphi$ and $d\eta_c/d\varphi$ are defined after differentiating equations (1). The coordinates of the instantaneous center (pole) P in coordinate system $O_e\xi\eta$ are:

$$\eta_P = r_{\omega_e} \cos\psi; \qquad \xi_P = r_{\omega_e} \sin\psi, \tag{13}$$

where $r_{\omega_e} = m\,N_e\,\lambda/2$ is a radius of the initial circle of epicycloidal gear.
Substituting equations (13), into equation (12) yields the following:

$$\frac{0.5m[N_1\cos\varphi + \lambda\cos(N_1\varphi) - \lambda N_2\cos\psi]}{0.5m[N_1\sin\varphi - \lambda\sin(N_1\varphi) - \lambda\sin\psi]} = \frac{0.5mN_1[\cos\varphi - \lambda\cos(N_1\varphi)]}{0.5mN_1[\sin\varphi - \lambda\sin(N_1\varphi)]}$$

(14)

Simplifying equation (14) yields equation (9), which shows that the two approaches are equally successfull in defining the tooth profile.

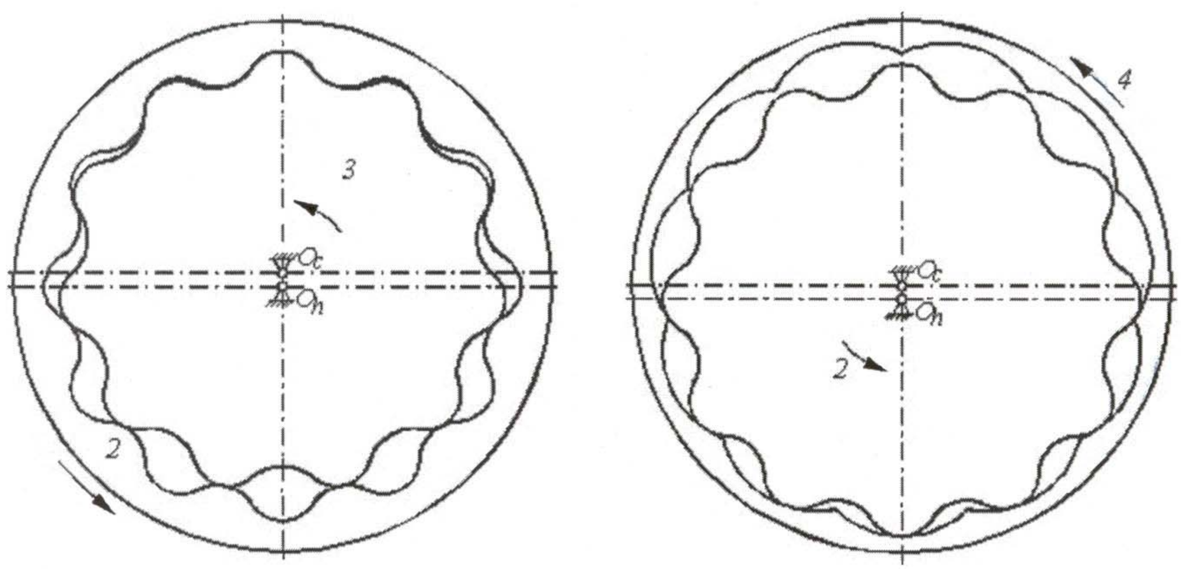

Fig. 6 Two Gearsets with m = 5 mm, $N_1 = 11$, $N_3 \equiv N_4 = 12$, x = 0.2 and $r_c^* = 1$

Fig.6 shown the two epicycloidal gearsets with m=5 mm, $N_1=N_3=N_4=11$, $N_2= 12$, x = 0.2 and $r_c^* = 1$. The left and the right tooth profile of gear *3* in gearset *3-2* represent a smooth curve, with no undercutting while the left and the right tooth profile generated for gear *4* in gearset *2-4* is undercut and doubles over itself. The result is, after teeth generation, that tooth profile is pointed and formed only in region A_4R_4 (Fig.5). Despite of this undercut and unfavorable pressure angles, gearset *2-4* is of great interest, because gear 2 with external teeth has one tooth more than gear 4 with internal teeth, so the pinion becomes the gear and the gear becomes the pinion.

4. Conclusions

The article offers a classification generated hypocycloidal gearsets with internal meshing where a difference in tooth numbers is one. It has been shown that, besides a compound hypocycloidal gear with cylindrical teeth the two hypocycloidal gearsets are possible where the compound gear wheel is replaced by an integral gear. The tooth profile of the gears of the two gearsets is formulated analytically. The results obtained by use of the conjugate action and the theory of envelopes and those obtained by use of the instantaneous center of velocities and the fundamental law of gearing proved to be the same.

It is of interest to point out a new gearset *2-4*, consisting of a hypocycloidal gear 2 with external teeth and pinion *4* with internal teeth. Due to the undercutting of tooth profiles in gear *4*, this gearset is characterized by unfavorable pressure angles and could be used only for a kinematic transmission but not for power transmission. The fact that the gear with external teeth has one tooth more than the internal gear is a unusual phenomenon in the theory of toothed wheels.

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MESHING CHARACTERISTICS OF HYPOCYCLOIDAL GEARS

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1. Abstract

The paper deals with kinematic characteristics of hypocycloidal gearsets generated by conjugate action of a known pinion with cylindrical teeth. The number of teeth of pinion is for one les than the number of teeth of gear. The line of contact of teeth is analytically formulated and its trends discussed in detail. The effects of the gearset parameters upon the locus of contact points are analytically determined and graphically plotted for the three typical gearsets.

2. Introduction

Cyloidal gearsets are characterized by a large gear ratio when they are designed in a planetary arrangement. The large reductions of input velocity (10 to 50 times) result from the difference of one tooth in numbers of teeth of the pinion and the internal gear. The sizes of the cycloidal gearsets are relatively smaller than those with other shape of tooth profile, including the involute profile, for the same gear ratios and the load ratings. Besides, the loss of power due to friction in these gearsets is reduced to a minimum. These characteristics of cycloidal gears have lead to their increased application in engineering practice [1].

A gearset containing a compound pinion with cylindrical teeth and an internal gear meshing together is shown in Fig.1 Tooth numbers of gear and pinion differ by one.

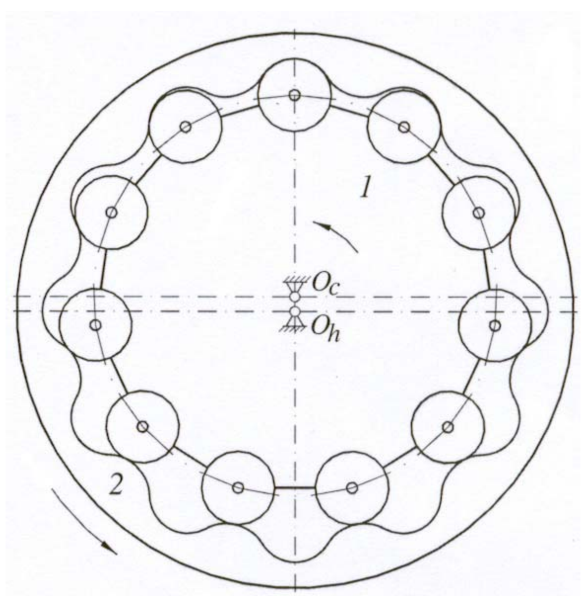

Fig. 1 Gearset of Pinion with Cylindrical Teeth and Epicycloidal Gear with m= 5 mm, $N_1 = 11$, $N_2 = 12$, x = 0.2 and $r_c^* = 1$

In the previous work [3] it has been shown that, besides this gearset, the other two gearsets can be generated, where compound pinion with cylindrical teeth has been replaced by an integral pinion with hypocycloidal external teeth. In this set the tooth profile of the pinion is generated using the conjugate action and external and internal envelope curve of the successive positions of the tooth profile of the hypocycloidal gear in the plain perpendicular to exes of the cenrodes that roll on each other with no slippage.
The objective can be achieved either by use of the theory of envelopes or by utilizing the law of gearing that assumes the instantaneous center of the pinion and gear to be fixed on the line of centers in order to have a constant angular velocities ratio.

In this article the authors aim to investigate the characteristics of meshing of the two hypocycloidal gearsets generated in the above mentioned manner. The locus of contact points is determined for each gearset and the effect of gear parameters upon the line of contact are analytically formulated and discussed. Also the loci of contact points are graphed for the three typical gearsets in Fig. 5.

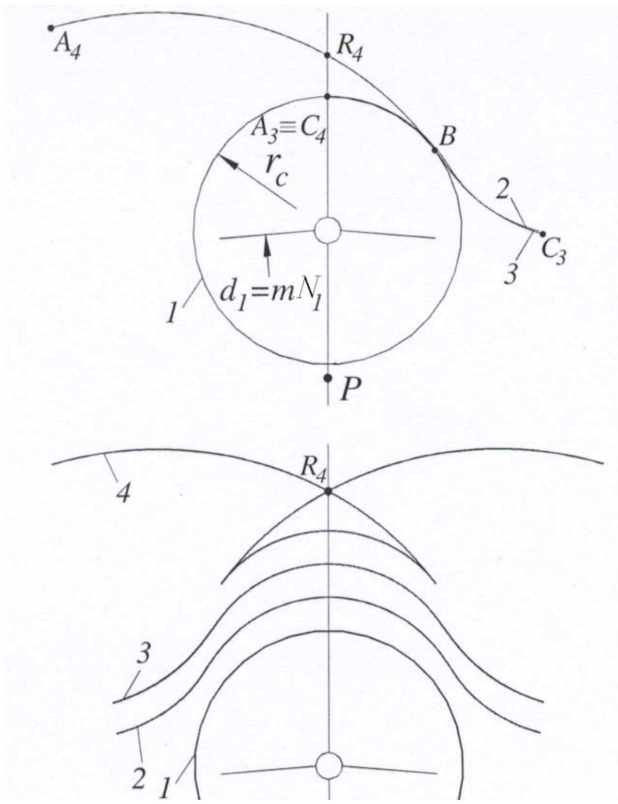

Fig. 2 Conjugate Tooth Profiles of Gearsets 1-2, 3-2 and 4-2 with N_1=N_3=N_4=11, N_2=12, x = 0.2 and $r_c^* = 1$

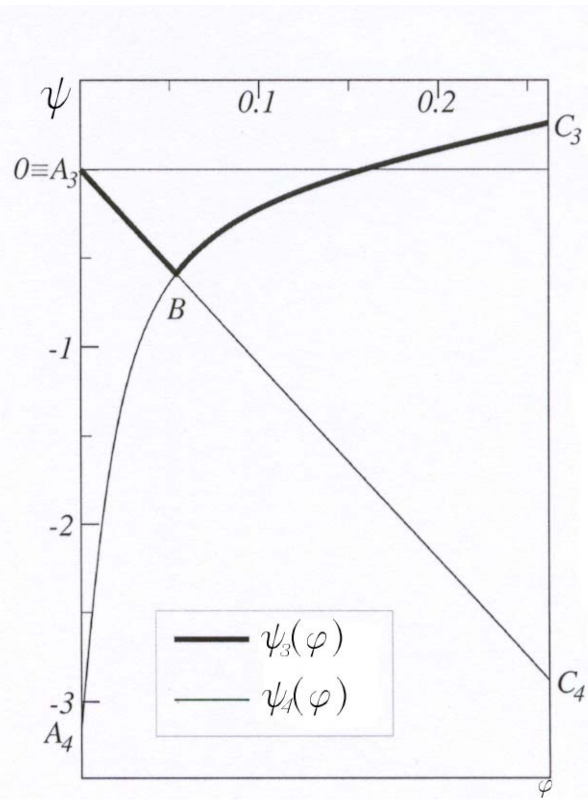

Fig 3 Relation Between Parameters ψ and φ for N_1=N_3=N_4=11, N_2=12 and x = 0.2

3. Determination of Line of Contact

Equations of the tooth profile of hypocycloidal wheel 2 for a typical hypocycloidal gearset are given by:

$$\xi_2 = \frac{m}{2}[(N_2 - 1)\sin\varphi - (1-x)\sin(N_2-1)\varphi + 2r_c^* \frac{(1-x)\sin(N_2-1)\varphi + \sin\varphi}{\sqrt{1-2(1-x)\cos N_2\varphi + (1-x)^2}}] \quad (1a)$$

$$\eta_2 = \frac{m}{2}[(N_2 - 1)\cos\varphi - (1-x)\cos(N_2-1)\varphi + 2r_c^* \frac{(1-x)\cos(N_2-1)\varphi + \cos\varphi}{\sqrt{1-2(1-x)\cos N_2\varphi + (1-x)^2}}] \quad (1b)$$

where *m, N, x* and r_c*=*1* are module, number of teeth, coefficient of modification (withdrawal) and coefficient of the generating circle radius respectively. The parameter φ varies within interval $[0,\pi/N_2]$. Pinion wheel *1* is of a compound design. It consists of a cylindrical hub and $N_1 = N_2 - 1$ cylinrical teeth of radius $r_c = m\, r_c^*$ with centres located on a circle with a radius $r = mN_1$. The centre distance is equal to $a_w = 0.5m(1 - x)$.

Two new hypocycloidal gearsets are generated by the pinion with cylindrical teeth, and they consist of hypocycloidal gear *2* with internal teeth , and pinion *3* with external teeth. The hypocycloidal wheel *2* with external teeth and gear *4* with internal teeth as per Fig.2. The tooth profile of the wheels replacing the wheel with cylindrical teeth of the same number of teeth is described in terms of parametric equations:

$$x_{3,4} = \frac{m}{2}[N_1\sin(\varphi + \frac{\psi_{3,4}}{N_1}) - \lambda\sin(N_1\varphi - \frac{\psi_{3,4}}{N_1}) - \lambda\sin(\frac{N_2}{N_1}\psi_{3,4}) + 2r_c^* \frac{\lambda\sin(N_1\varphi - \frac{\psi_{3,4}}{N_1}) - \sin(\varphi + \frac{\psi_{3,4}}{N_1})}{\sqrt{1-2\lambda\cos N_2\varphi + \lambda^2}}] \quad (2a)$$

$$y_{3,4} = \frac{m}{2}[N_1\cos(\varphi + \frac{\psi_{3,4}}{N_1}) - \lambda\cos(N_1\varphi - \frac{\psi_{3,4}}{N_1}) - \lambda\cos(\frac{N_2}{N_1}\psi_{3,4}) - 2r_c^* \frac{\lambda\cos(N_1\varphi - \frac{\psi_{3,4}}{N_1}) - \cos(\varphi - \frac{\psi_{3,4}}{N_1})}{\sqrt{1-2\lambda\cos N_2\varphi + \lambda^2}}] \quad (2b)$$

where $\lambda = 1 - x$ is a factor les than 1, and ψ is the angle of rotation of the hypocycloidal wheel about instantaneous center of velocitiy P. The angle ψ is defined by equation:

$$\psi_{3,4}(\varphi) = \varphi + \cos^{-1}\frac{-\lambda\sin^2 N_2\varphi \mp \sqrt{\lambda^2\sin^4 N_2\varphi - [\lambda^2 - 2\lambda\cos N_2\varphi + 1][2\lambda - (1+\lambda^2)\cos N_2\varphi]\cos N_2\varphi}}{\lambda^2 - 2\lambda\cos N_2\varphi + 1} - \pi \quad (3)$$

The locus of contact points is described by parametric equations:

$$x_k = x_{3,4}\cos(\frac{N_2\psi_{3,4}}{N_1}) - y_{3,4}\sin(\frac{N_2\psi_{3,4}}{N_1}) \quad (4a)$$

$$y_k = x_{3,4}\sin(\frac{N_2\psi_{3,4}}{N_1}) + y_{3,4}\cos(\frac{N_2\psi_{3,4}}{N_1}) \quad (4b)$$

It is obvious that the tooth-profiles of the wheels 3 and 4, relative to points of contact depend on parameters N_1, x and r_c^*. In order to define the effect of these parameters, it is necessary to examine the relation (3) between the parameters ψ and φ. It can be concluded:

$$\psi_3(\varphi) = -N_1\varphi, \qquad \varphi \in \left[0, \frac{\cos^{-1}(\lambda)}{N_2}\right] \quad (5a)$$

$$\psi_3(\varphi) = \varphi + \cos^{-1}\frac{-\lambda\sin^2 N_2\varphi + (\lambda - \cos N_2\varphi)(\lambda\cos N_2\varphi - 1)}{\lambda^2 - 2\lambda\cos N_2\varphi + 1} - \pi; \quad \varphi \in \left[\frac{\cos^{-1}(\lambda)}{N_2}, \frac{\pi}{N_2}\right] \quad (5b)$$

$$\psi_4(\varphi) = \varphi + \cos^{-1}\frac{-\lambda\sin^2 N_2\varphi + (\lambda - \cos N_2\varphi)(\lambda\cos N_2\varphi - 1)}{\lambda^2 - 2\lambda\cos N_2\varphi + 1} - \pi; \qquad \varphi \in \left[0, \frac{\cos^{-1}(\lambda)}{N_2}\right] \quad (5c)$$

$$\psi_4(\varphi) = -N_1\varphi, \qquad \varphi \in \left[\frac{\cos^{-1}(\lambda)}{N_2}, \frac{\pi}{N_2}\right] \quad (5d)$$

Obviously within the range $\varphi \in [0, \cos^{-1}(\lambda)/N_2]$ function $\psi_3(\varphi)$ decreases linearly. The same is true for function $\psi_4(\varphi)$ within the range $\varphi \in [\cos^{-1}(\lambda)/N_2, \pi/N_2]$. In order to examine the trend of change of function $\psi_3(\varphi)$ within the range $\varphi \in [\cos^{-1}(\lambda)/N_2, \pi/N_2]$, and of function $\psi_4(\varphi)$ within the range $\varphi \in [0, \cos^{-1}(\lambda)/N_2]$, it is necessary to check the derivative:

$$\frac{d\psi_3(\varphi)}{d\varphi} = \frac{d\psi_4\varphi)}{d\varphi} = \frac{d[\varphi + \cos^{-1}\frac{-\lambda\sin^2 N_2\varphi + (\lambda - \cos N_2\varphi)(\lambda\cos N_2\varphi - 1)}{\lambda^2 - 2\lambda\cos N_2\varphi + 1} - \pi]}{d\varphi} \quad (6)$$

After few rearrangements one can obtain the following:

$$\frac{d\psi(\varphi)}{d\varphi} = 1 - \frac{N_2(\lambda^2 - 1)}{\lambda^2 - 2\lambda\cos N_2\varphi + 1} \quad ; \quad \varphi \in [0, \frac{\pi}{N_2}]$$

(7)

Since $N_2(\lambda^2 - 1) < 0$, and $\lambda^2 - 2\lambda\cos(N_2\varphi) + 1 > 0$ for ($\lambda \in (0,1)$), one can conclude that $d\psi(\varphi) > 1 > 0$, which means that the function $\psi_3(\varphi)$ within the range $\varphi \in [\cos^{-1}(\lambda)/N_2, \pi/N_2]$ as well as the function $\psi_4(\varphi)$ within the range $\varphi \in [0, \cos^{-1}(\lambda)/N_2]$ are monotonically nondecreasing functions.

The function $\psi_3(\varphi)$ has three extremes as shown in Fig.3. In the range $\varphi \in [0, \cos^{-1}(\lambda)/N_2]$ the function is monotonically decreasing. At $\varphi = 0$ the function has a local maximum equal to zero and at the end of the range at $\varphi_B = \cos(\lambda)/N_2$, it reaches a global minimum equal to $-\cos(\lambda)N_1/N_2$. Within the range $\varphi \in [\cos^{-1}(\lambda)/N_2, \pi/N_2]$ the function is increasing and at the end of the range at value $\varphi = \pi/N_2$ the function reaches a global maximum equal to π / N_2.

The function $\psi_4(\varphi)$ also has three extremes as shown in Fig 3.. In the range $\varphi \in [0, \cos^{-1}(\lambda)/N_2]$ the function is a monotonically nondecreasing function. At $\varphi = 0$ the function has a global minimum equal to $-\pi$. At $\varphi_B = \cos^{-1}(\lambda)/N_1)$ the function reaches a global maximum equal to $-\cos^{-1}(\lambda)N_1/N_2$. Within the range $\varphi \in [\cos^{-1}(\lambda)/N_2), \pi/N_2]$ the function is a monotonically nonincreasing and at $\varphi = \pi/N_2$ it reaches a local minimum equal to $\pi N_1/N_2$.

Fig.4 shows the lines of contact of two hypocycloidal gears with m = 5 mm, $N_1 = 11$, $r_c^* = 1$ and $x = 0.2$. Portion of contact line A_3BC_4 corresponding to a linear relation between the parameters ψ and φ, in both epicycloidal gears is equal to the line of contact of the pinion with cylindrical teeth and the hypocycloidal internal gear. Portion of contact line BC_3 of gerset *3-2* corresponds to line BC_3 of function $\psi_3(\varphi)$ and part BA_4 of gearset *2-4* corresponds to line BA_4 of function $\psi_4(\varphi)$. Gearset *3-2* exhibits two contact points within the range as shown in Fig.4 (a). When $\psi_3(\varphi)$ is changed within the range $\psi_{C3} = \pi/N_4$ to zero one contact point exists, while within the range from zero to $\psi_B = -\cos^{-1}(\lambda)N_1/N_2$ two contact points occur. Within the range of $\psi_B = -\cos^{-1}(\lambda)N_1/N_2$ to $-\pi N_1/N_2$ only one contact point exists.

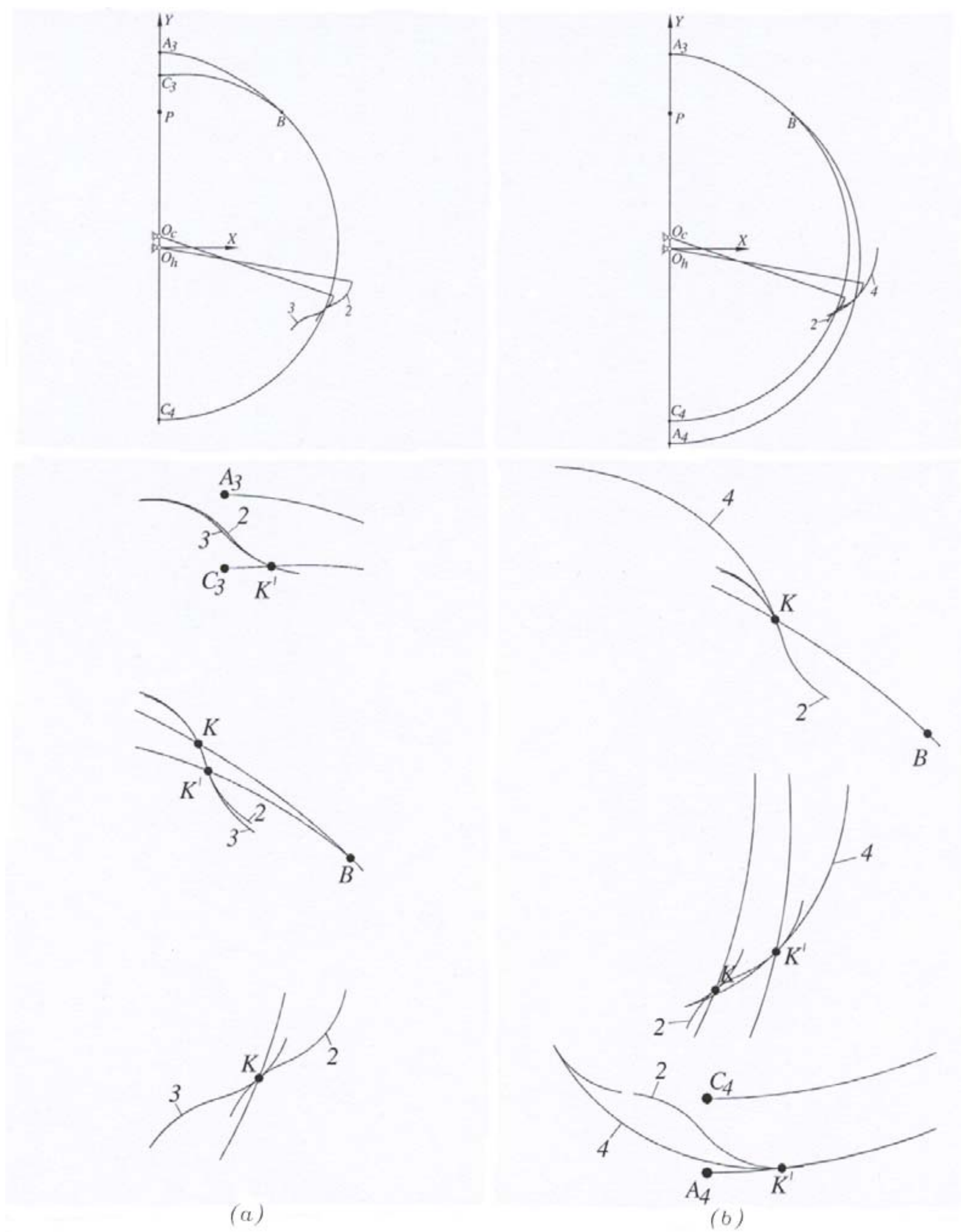

Fig. 4 Lines of Contact of Two Hypocycloidal Gearsetss with m = 5 mm, N_1 = 11, x = 0.2 and r_c^* =1

In gearset *2-4* $\psi_4(\varphi)$ also passes through contact ranges with one and two contact points as shown in Fig.4.(b). When $\psi_4(\varphi)$ changes from zero to $\psi_B = -\cos^{-1}(\lambda)N_1/N_2)$ one contact point exists. Within the range from $\psi_B = -\cos^{-1}(\lambda)N_1/N_2$ to $-\pi N_1/N_2$ two contact points occur. Within the range from $-\pi N_1/N_2$ to π only one contact point exists. Due to the fact that the profile of gear wheel 4 makes a loop which is undercut so that only part RA_4 remains, there is only one point of contact.

Fig. 5 shows the loci of contact points drown by a continuous lines for a gearset consisting of a compound pinion with cylindrical teeth and a hypocycloidal pinion and the two hypocycloidal gear generated with m = 5 mm, N_1 = 11, x = 0.2 and r_c*=1.

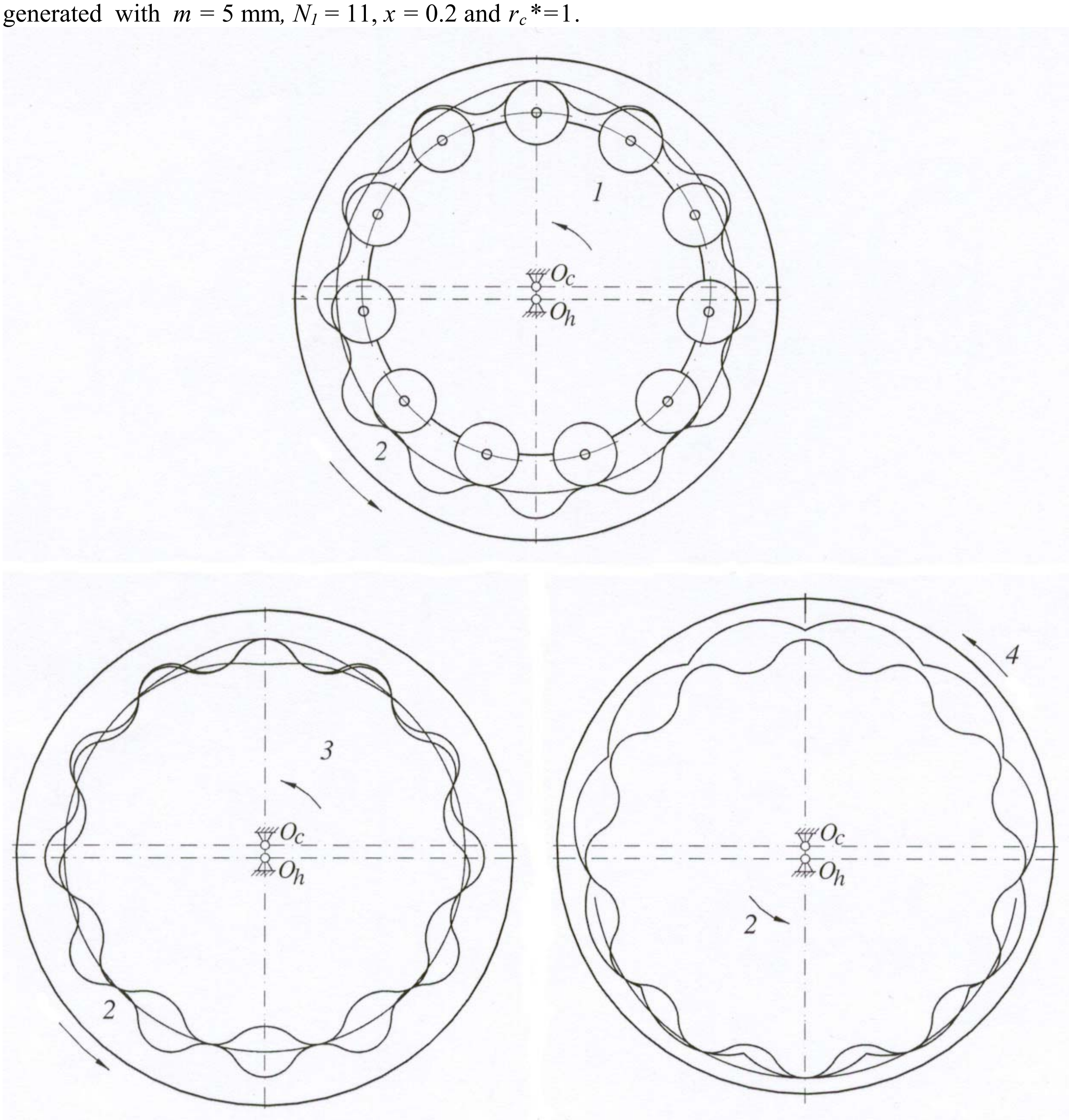

Fig. 5 Lines of Contact for Three Hypocycloidal Gearsets with
m = 5 mm, N_1= 11, N_2= 12, x = 0.2 and r_c^*=1

4. Conclusions

The objective of the study that had been set to generate and analytically formulate parameters of the gearsets based on a pinion with cylindrical tooth profile using the conjugate action and replace the pinion with an integral wheel with hypocycloidal tooth profile has been accomplished.. The line of contact has been formulated analytically. The characteristic portions of the line of contact have been critically analyzed and the effect of geaset basic parameters on the line of contact have been established.

As the most important conclusion from the results of this analysis is that generated gearset *3-2* would have an increased load rating because the contact of a convex and a concave surfaces occurs that should yield a lower Hertzian contact stress. In addition a multiple contact points occur along line of contact and an increased load rating could be claimed too. This aspect remains to be investigated by a contact stresses determination. From the property of cycloidal curves the curvature of the surfaces in contact are readily available.

Since the common normal from each contact point must pass through the instantaneous center it is obvious that the pressure angle vary and the transmitted torque is expected to vary. Since there are a half of teeth in contact at any instant it is hoped that the torque will average to a value close to constant.

Gearset *2-4* resulting from the conjugate generating process represents an unusual phenomenon in the theory of the toothed wheels since the number of teeth of wheel 2 is for one larger than the number of teeth of wheel 4, hence designation of the pinion and the gear does not hold anymore. Due to unfavorable pressure angles, as result of undercutting of tooth profile of wheel 4 this gearset is unsuitable for power transmission. Yet, by itself, it is an interesting kinematic phenomenon

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