Input-Output Kinematic and Static Equations for Gripper Fingers Modeled as Planar Parallel Manipulators

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Abstract

This paper presents a method to determine the kinematic and static equations of a gripper finger using reciprocal screws. Gripper fingers are modeled as planar parallel manipulators with up to 3 degrees of freedom (dof).

1. Introduction

High manipulator dexterity can be achieved with multi-dof grippers. To minimize complexity, passive rather than active actuators can be used for some of the joints of the gripper [1]. The objective of this research is to perform a kinematic and static analysis on multi-dof gripper fingers.
The following assumptions are made for the finger mechanism:
(i) The finger has the kinematic chain of a general non-redundant planar parallel manipulator with up to three dof. It consists of a base and a platform connected by up to three distinct serial subchains (legs). The legs may have different numbers of joints.
(ii) The actuated joints are chosen arbitrarily with no more than three non-actuated joints per leg.
(iii) The number of phalanges (links in contact with the object) is equal to the mobility of the finger.
(iv) Any moving link can be a phalanx.

2. Methodology

To perform a kinematic and static analysis we modify the method of reciprocal screws used in [2] for the analysis of n-dof parallel mechanisms. In this method, the platform twist, is expressed as a linear combination of the joint twists in each leg. Then, the non-actuated joint twists are eliminated by multiplying (using the reciprocal screw product) the twist equation for each leg with a reciprocal screw. If the reciprocal screws are chosen correctly, i.e. as a maximal linearly independent set [2], the resulting input-output velocity equation is a necessary and sufficient condition for the feasibility of the instantaneous motion of the mechanism.
The study of a gripper finger differs from the analysis of a parallel mechanism in two ways: (i) the legs may contain different numbers of joints; and (ii) the output is the motion of the phalanx which may be any moving link, not necessarily the platform. For a gripper, the instantaneous kinematics is described by expressing the 3-dimensional planar phalanx twist, $\boldsymbol{\xi}$, as a linear combination of the joint twists in each of the k serial subchains that connect the phalanx with the base via the k different legs of the parallel chain:

$$\boldsymbol{\xi} = \sum_{i \in L_j} \boldsymbol{\xi}_i \dot{\theta}_i \,, \quad j = 1, \ldots, k \,, \tag{1}$$

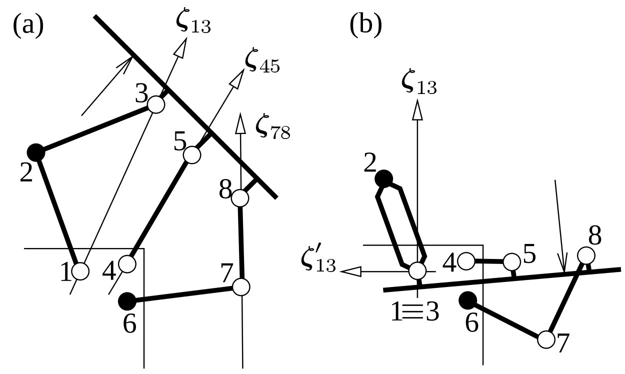

Figure 1: Case 1 (a) non-singular configuration (b) singular configuration

where $\dot{\theta}_i$, $\boldsymbol{\xi}_i$, $i = 1, 2, 3 \ldots$ are, respectively, the joint velocities and joint screws, while L_j is the subset of values of i corresponding to the joints in the j-th subchain. When the phalanx is a link different from the moving platform the subchains, and therefore the sets L_j, are not non-intersecting. The objective is to eliminate the non-actuated joint twists from (1) and obtain an input-output equation relating the twist of the phalanx and the input velocities.

3. Analysis

Given assumptions $(i)-(iv)$ for the gripper finger, it can be shown that the analysis can be performed as in one of the following three cases.

Case 1. There are two or fewer non-actuated joint screws per serial subchain from base to phalanx. For each subchain we choose a maximum collection of linearly independent reciprocal screws [2]. The product of these reciprocal screws with the twist equation of the subchain will contain no non-actuated joint velocities. The resulting system of input-output equations is a necessary and sufficient condition for the feasibility of the velocities. It will consist of exactly three equations unless the mechanism is in a special singular configuration where two non-actuated joint twists in a subchain are linearly dependent.

Example 1. Fig. 1(a) illustrates a 2-dof finger with three serial subchains. The actuated joints are filled in black. The phalanx is specified by a pointer. The three instantaneous twist equations are defined by the three serial subchains (1, 2, 3), (4, 5) and (6, 7, 8), respectively. The reciprocal screws $\boldsymbol{\zeta_{13}}$, $\boldsymbol{\zeta_{45}}$ and $\boldsymbol{\zeta_{78}}$ shown in Fig. 1(a) are used to eliminate the non-actuated joint twists in each serial subchain. In matrix

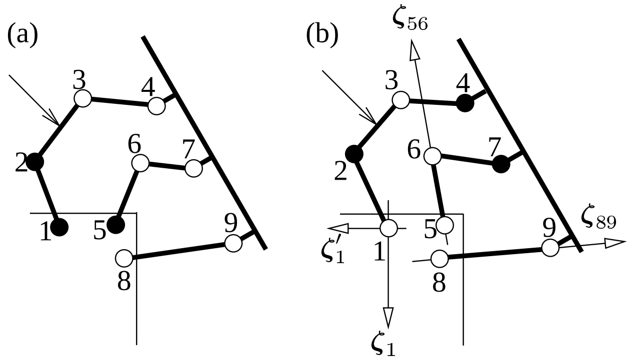

Figure 2: (a) Case 2 (b) Case 3

form, the input-output velocity equations become:

$$\begin{bmatrix} \boldsymbol{\zeta_{13}}\circ \\ \boldsymbol{\zeta_{45}}\circ \\ \boldsymbol{\zeta_{78}}\circ \end{bmatrix} \boldsymbol{\xi} = \begin{bmatrix} \boldsymbol{\zeta_{13}} \circ \boldsymbol{\xi}_1 & 0 & 0 \\ 0 & \boldsymbol{\zeta}_{78} \circ \boldsymbol{\xi}_6 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_6 \\ 0 \end{bmatrix} ,$$

where "$\circ$" denotes the reciprocal screw product.
In Fig. 1(b), joints 1 and 3 become coincident. Since the non-actuated joint twists form a 1-system, we must select *two* linearly independent reciprocal screws, $\boldsymbol{\zeta_{13}}$ and $\boldsymbol{\zeta_{13}'}$ passing through point 1≡3. This will generate an additional, fourth velocity equation:

$$\boldsymbol{\zeta_{13}'} \circ \boldsymbol{\xi} = \boldsymbol{\zeta_{13}'} \circ \boldsymbol{\xi_2}\dot{\theta}_2 \,.$$

Case 2. One subchain contains no non-actuated joint screws and the other subchains contain three or more non-actuated joint twists. The twist equation of the first serial subchain contains only actuated joint velocities and therefore provides three input-output velocity equations. These equations completely describe the instantaneous kinematics of the finger unless in one of the other subchains all non-actuated joint screws belong to a 2-system. In such a singular configuration an additional equation is needed to describe the input and output. This equation is obtained by eliminating all non-actuated joint twists in the serial subchain by multiplication with a reciprocal screw (such a screw exists since the non-actuated twists span only a 2-system).

Example 2. Fig. 2(a) illustrates a mechanism having 2 serial subchains each containing exactly 4 non-actuated joint screws. The three input-output equations are generated by the subchain (1,2) containing only actuated joint screws:

$$\boldsymbol{\xi} = \boldsymbol{\xi}_1\dot{\theta}_1 + \boldsymbol{\xi}_2\dot{\theta}_2 \,.$$

Case 3. The chain has three legs. One subchain from phalanx to base has one non-actuated joint twist and the other two subchains have exactly three such joint twists. The first serial subchain generates two linearly independent equations. The third equation can be generated via an elimination technique from the twist equations of the other two subchains.
Assumption (ii) implies that there exists a non-actuated joint screw common to the second and third serial subchains. We select reciprocal screws that eliminate the four non-actuated joint screws not common to both subchains. The result is a system of two scalar equations containing only one non-actuated joint velocity. This velocity is eliminated by taking a linear combination of the two equations.

Example 3. The mechanism in Fig. 2(b) has two serial subchains each with exactly three non-actuated joint screws. The reciprocal screws $\boldsymbol{\zeta_1}$ and $\boldsymbol{\zeta_1'}$ are used to eliminate the non-actuated joint screw in the subchain $(1, 2)$.
Joint screw 3 is common to both remaining subchains. The reciprocal screw $\boldsymbol{\zeta_{56}}$ is used to eliminate the two non-actuated joint screws 5 and 6 while the screw, $\boldsymbol{\zeta_{89}}$ eliminates the two non-actuated joint screws 8 and 9. The following two equations result:

$$\boldsymbol{\zeta_{56}} \circ \boldsymbol{\xi} = \boldsymbol{\zeta_{56}} \circ \boldsymbol{\xi_3}\dot{\theta}_3 + \boldsymbol{\zeta_{56}} \circ \boldsymbol{\xi_4}\dot{\theta}_4 + \boldsymbol{\zeta_{56}} \circ \boldsymbol{\xi_7}\dot{\theta}_7 \quad (2)$$

$$\boldsymbol{\zeta_{89}} \circ \boldsymbol{\xi} = \boldsymbol{\zeta_{89}} \circ \boldsymbol{\xi_3}\dot{\theta}_3 + \boldsymbol{\zeta_{89}} \circ \boldsymbol{\xi_4}\dot{\theta}_4 \,. \quad (3)$$

Now we multiply (2) by c_1 and (3) by c_2 and add. By choosing $c_1 = \boldsymbol{\zeta_{89}} \circ \boldsymbol{\xi_3}$ and $c_2 = -\boldsymbol{\zeta_{56}} \circ \boldsymbol{\xi_3}$, $\dot{\theta}_3$ is eliminated. The input-output equation becomes:

$$\begin{bmatrix} \boldsymbol{\zeta_1}\circ \\ \boldsymbol{\zeta_1'}\circ \\ c_1\boldsymbol{\zeta_{56}} \circ -c_2\boldsymbol{\zeta_{89}}\circ \end{bmatrix} \boldsymbol{\xi} = \begin{bmatrix} \boldsymbol{\zeta_1} \circ \boldsymbol{\xi_2} & 0 & 0 \\ \boldsymbol{\zeta_1'} \circ \boldsymbol{\xi_2} & 0 & 0 \\ 0 & a & b \end{bmatrix} \begin{bmatrix} \dot{\theta}_2 \\ \dot{\theta}_4 \\ \dot{\theta}_7 \end{bmatrix},$$

where $a = c_1\boldsymbol{\zeta_{56}} \circ \boldsymbol{\xi_4} - c_2\boldsymbol{\zeta_{89}} \circ \boldsymbol{\xi_4}$ and $b = c_1\boldsymbol{\zeta_{56}} \circ \boldsymbol{\xi_7}$.

4. The General Input-Output Equation

According to the analysis in Section 3, in all three cases, the input-output equation can be expressed in a general matrix form:

$$\mathbf{R}\boldsymbol{\xi} = \boldsymbol{\Lambda}\dot{\boldsymbol{\theta}} \,,$$

where $\dot{\boldsymbol{\theta}}$ is the vector of actuated joint velocities, possibly augmented with zeroes as in Example 1, while $\mathbf{R}$ and $\boldsymbol{\Lambda}$ are $3{\times}3$ Jacobian matrces. The corresponding static equation is:

$$\mathbf{w} = \mathbf{R}^T(\boldsymbol{\Lambda}^{-1})^T\mathbf{f} \,,$$

where $\mathbf{w}$ is the resultant wrench at the contact point, and $\mathbf{f}$ is the matrix of the joint torques.

5. Conclusions

Three cases arise in the kinematic and static analysis of a gripper finger with the architecture of a planar parallel manipulator and arbitrary placement of the phalanges and actuators. In each case the non-actuated joint velocities can be eliminated from the equations of the instantaneous kinematics. As a result an input-output velocity equation is obtained.

References

[1] Montambault, S. and Gosselin, C. M., 1998. "Analysis of Underactuated Mechanical Grippers", to appear in *ASME Journal of Mechanical Design.*

[2] Zlatanov, D. S., Fenton, R. G., and Benhabib, B.,1994. "Analysis of the Instantaneous Kinematic and Singular Configurations of Hybrid-Chain Manipulators". *ASME 23rd Mechanisms Conference.* Vol. DE-72, pp 467-476.